About the Book:

The PG and Research Department of Mathematics, Saiva Bhanu Kshatriya College, Aruppukottai has organized a National Seminar on Current Perspectives in Mathematics on 01.04.2023, to focus on the recent innovative developments in the field of Mathematics. The Proceedings of the Seminar is duly edited and to be brought out by Dr. N. Kandaraj, Associate Professor and Head PG and Research Department of Mathematics and by Dr. V. Thiruveni, Assistant Professor of the same department. This edited book volume contains 19 articles belonging to various fields in Mathematics.

₹ Rs. 400 /-

ISBN 978-93-92504-22-8

9 789392 504228

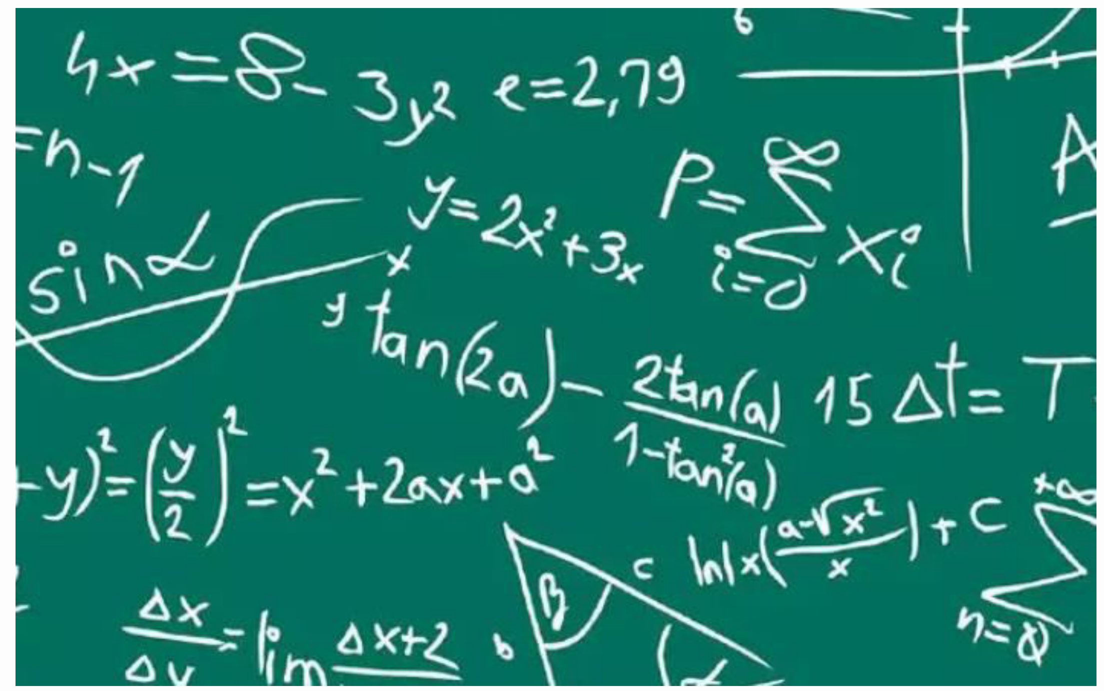
Research Culture Society and Publication
An International ISBN Book Publisher
www.researchculturesociety.org

PROCEEDINGS OF THE NATIONAL SEMINAR ON CURRENT PERSPECTIVES IN MATHEMATICS

PG and Research Department of Mathematics
Saiva Bhanu Kshatriya College,
Aruppukottai, Tamil Nadu, India

உயர்வு உள்க

PROCEEDINGS OF THE NATIONAL SEMINAR ON CURRENT PERSPECTIVES IN MATHEMATICS



Edited By:
Dr. N. KANDARAJ
Dr. V. THIRUVENI

Research Culture Society and Publication
www.researchculturesociety.org

PROCEEDINGS OF THE NATIONAL SEMINAR ON CURRENT PERSPECTIVES IN MATHEMATICS

EDITORS

1. Dr. N. KANDARAJ (Convener),
Associate Professor and Head,
PG and Research Department of Mathematics
Saiva Bhanu Kshatriya College, Aruppukottai
E-Mail: n.kandarajsbkc1998@gmail.com

2. Dr. V. THIRUVENI (Organizing Secretary),
Assistant Professor,
PG and Research Department of Mathematics
Saiva Bhanu Kshatriya College, Aruppukottai
E-Mail: thiriveni2009@gmail.com

Organized by

PG and Research Department of Mathematics
Saiva Bhanu Kshatriya College
Aruppukottai, 626101, India

ISBN : 978-93-92504-22-8

Published by :

Research Culture Society and Publication

www.researchculturesociety.org

PROCEEDINGS OF THE NATIONAL SEMINAR ON CURRENT PERSPECTIVES IN MATHEMATICS

EDITED BY :

Dr. N. KANDARAJ
Dr. V. THIRUVENI

Published and Printed at : **(First Edition – April, 2023)**

Research Culture Society and Publication / Research Culture Society
(Reg. International ISBN Books and ISSN Journals Publisher)
India : C – 1, Radha Raman Soc, At & Po - Padra, Dis - Vadodara, Gujarat, India – 391440.
USA : 7886, Delrosa Avenue, Sanbernardino, CA 92410.
Canada : Loutit Road, Fort McMurray, Alberta, T9k0a2.
Greece : Mourkoussi Str, Zografou, Athens, 15773

Email: RCSPBOOKS@gmail.com / editor@ijrcs.org
www.researchculturesociety.org / www.ijrcs.org

MRP : Rs. 400 /-

ISBN : 978-93-92504-22-8

ISBN 978-93-92504-22-8
9 789392 504228

Research Culture Society and Publication

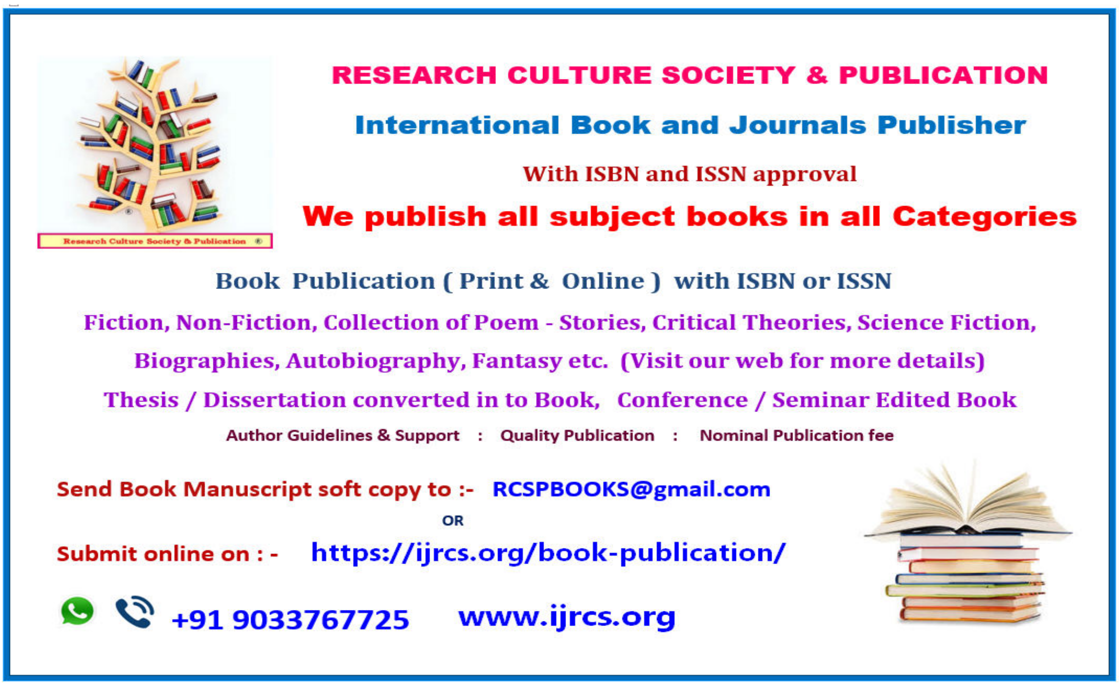
(Reg. International ISBN Books and ISSN Journals Publisher)

Email: RCSPBOOKS@gmail.com / editor@ijrcs.org

WWW.RESEARCHCULTURESOCIETY.ORG / WWW.IJRCS.ORG

Conference, Seminar, Symposium organization in association/collaboration with different Institutions.

Conference, Seminar, Symposium Publication with ISSN Journals and ISBN Books (Print / Online).



About the Seminar

The National Seminar on Current Perspectives in Mathematics was conducted by the PG and Research Department of Mathematics on 01.04.2023 in the New Conference Hall, Saiva Bhanu Kshatriya College, Aruppukottai. 163 participants, including students, research scholars and faculty members, attended the Seminar. The theme of the seminar was to motivate the students, researchers and young faculty to develop interest towards higher education and research in Mathematics. Dr. M. Chandramouleeswaran, Head (Retired), PG and Research Department of Mathematics, S.B.K. College delivered the Keynote address, which motivated the participants towards the technical sessions.

In the technical session I, Dr. R. Kala, Professor, M. S. University, Tirunelveli delivered a lecture on the topic "*Some new graph Parameters*". The participants got the idea about the developing concepts in the field of graph theory. They enjoyed the session and interacted with the resource person. In the technical session II, Dr. S. Rajeshwari, Assistant Professor, BIT, Bangalore delivered a lecture on the topic "*Complex Analysis and Value Distribution Theory*". The talk was well organized and it gave the participants, clear information about the Value Distribution Theory. In the technical session III, Dr. M. Chandramouleeswaran recalled the definitions of semigroups and semirings and explained how a semiring valued semigraph was constructed, in his talk on "*Semiring Valued Semi-graphs*". On the whole, all three sessions gave a platform for the participants to interact with reputed Resource Persons.

The paper presentation was conducted in 4 parallel sessions. 29 participants presented their papers in the seminar. From them, 19 papers were shortlisted by the Editors for publication in the Proceedings.

About the College

Saiva Bhanu Kshatriya College, Aruppukottai, Tamil Nadu, India was established by Aruppukottai Nadar Uravinmurai Pothu Abiviruthi Trust in 1970. The college is an aided co-educational institution affiliated to Madurai Kamaraj University, Madurai. The college offers UG and PG Courses in various disciplines. Two departments are upgraded as Research Centre. The vision of the college is to impart quality higher education to the socio-economically weaker student community. The aim of the college is “Aim High” reflecting the pursuit of excellence. The college also provides value-based education and train the students to become worthy citizens.

About the Department

The department of Mathematics was established in the year 1970 with the Pre University Course. B.Sc. (Mathematics) and M.Sc. (Mathematics) were introduced in 1982 and 1987 respectively. In the year 2013, it became a research centre. Active research is being carried on by the faculty of the department in both pure and applied mathematics. The department is conducting National Seminar every year with funds from various funding agencies and also as self-funded. Through the department, 26 research scholars were awarded Ph.D. degree. The staff members of the department published more than 150 research papers in reputed, peer-reviewed National and International journals and presented many research papers in National and International Conferences.

Organizing committee

Patrons

Thiru. C.J. Gnana Gowthama Pandiyan,

President, S.B.K. College, Aruppukottai.

Thiru. R. Gunasekaran,

Secretary, S.B.K. College, Aruppukottai.

Thiru. G. Rathinasamy,

Vice President, S.B.K. College, Aruppukottai.

Thiru. M. Pragadish Prabu,

Assistant Secretary, S.B.K. College, Aruppukottai.

Thiru. R. Sundarajan,

Treasurer, S.B.K. College, Aruppukottai.

Thiru. N.R. Ramesh,

Member, S.B.K. College, Aruppukottai.

Chairman

Dr. K. Sudhagaran, M.Com., M.Phil., Ph.D.,

Principal (i/c), S.B.K. College, Aruppukottai..

Convener

Dr. N. Kandaraj, Head & Associate Professor,

Organizing Secretary

Dr. V. Thiruveni, Assistant Professor

Organizing Committee Members

Dr. M. Rajkumar, Associate Professor

Dr. K.P.V. Preethi, Assistant Professor

Dr. S. Raja Dharik, Assistant Professor

Mr. M. Kasi Mayan, Assistant Professor

Dr. K. Sujatha, Assistant Professor

Mrs. S. Kavitha, Assistant Professor

Mrs. P. Suseela, Assistant Professor

Mrs. P. Jainumbu Beevi, Assistant Professor

Message from the Principal

Dr. K. Sudhagaran,

Principal (i/c)

Saiva Bhanu Kshatriya College,

Aruppukottai - 626101.

Greetings to all the participants and faculty members of the PG and Research Department of Mathematics. The PG and Research Department of Mathematics, Saiva Bhanu Kshatriya College is functioning since the academic year 1971-1972 with teaching Mathematics for Pre-University Course. In the year 1982-1983 B.Sc., Mathematics course was introduced. The Department started offering M.Sc. Programme in Mathematics in the year 1987-1988. It also offers M.Phil., programme in Mathematics as a self-supporting programme since 2007-2008. It has been upgraded into a full-pledged research center from the academic year 2013-2014. Active research is being carried on by the faculty of the department in both pure and applied mathematics. The department is conducting National Seminar every year with funds from various funding agencies and also as self-funded. This year, they have conducted the National Seminar on Current Perspectives in Mathematics. The Proceedings of the Seminar is duly edited and brought out by Dr. N. Kandaraj, Associate Professor and Head of the Department and Dr. V. Thiruveni, Assistant Professor, PG and Research Department of Mathematics of our college as the Convener and Organizing Secretary of the Seminar. My best wishes to the faculty members of the department and the participants.

From the Desk of Editors

The PG and Research Department of Mathematics, Saiva Bhanu Kshatriya College, Aruppukottai has been organizing National Seminar every year to focus on the recent innovative developments on various fields of Mathematics. It paved a way for the research scholars and young college teachers to exchange information in different branches of Mathematics with the experts in the field. This year the National Seminar on Current Perspectives in Mathematics is organized on 01.04.2023. The seminar was inaugurated by Dr. M. Chandramouleeswaran, Retired Head of our Department under the president ship of Mr. R. Gunasekaran, Secretary, Saiva Bhanu Kshatriya College Managing Board, in the presence of Dr. K. Sudhagaran, Principal(i/c). Many distinguished mathematicians from various universities and colleges actively participated in the seminar. There were three invited speakers and 29 contributory papers. The topics discussed during the seminar include Graph Parameters, Complex Analysis and S-Valued Semi-graphs. This proceeding contains 19 selected papers presented by the participants at the seminar. We take this opportunity to thank the Managing Board of Saiva Bhanu Kshatriya College for their constant encouragement to organize the seminar and edit the proceedings. We extend our thanks to Dr. K. Sudhagaran, Principal (i/c), Saiva Bhanu Kshatriya College, Aruppukottai for his support and encouragement. Our special thanks to the staff members, research scholars and students of the Department for their enthusiastic and unstinted support rendered for the successful conduct of the Seminar. We thank all the participants but for whom the seminar would not have been such a success. Finally, we thank the Editor, Research Culture Society and Publication for his help in bringing this proceeding.

Dr. N. Kandaraj and Dr. V. Thiruveni
Editors

TABLE OF CONTENTS

PARTIAL WEIGHT DOMINATION NUMBER OF S-VALUED GRAPHS

P. Malathy[1] and V. Thiruveni[2]

[1] Research Scholar, PG and Research Department of Mathematics,
Saiva Bhanu Kshatriya College, (Affiliated to Madurai Kamaraj University, Madurai)
Aruppukottai, , Tamil Nadu, India. malathyselvaraj77@gmail.com

[2] Assistant Professor, PG and Research Department of Mathematics,
Saiva Bhanu Kshatriya College, (Affiliated to Madurai Kamaraj University, Madurai)
Aruppukottai, Tamil Nadu, India. thiriveni2009@gmail.com

ABSTRACT: *The theory of domination in graphs has been studied by several authors. In 2015, Chandramouleeswaran and others introduced semiring valued graphs (that is graphs whose vertices and edges are labeled with values from a semiring S) and they have done several works on weight dominating vertex sets and weight domination number for S-valued graphs. Motivated by these previous investigations, we work here on the partial weight domination number of S- valued graphs. We get interest in studying the relationship between the partial weight domination and the weight domination parameters. Some bounds on* (α, p) *partial weight domination number of* G^S*, that is* $\gamma_{(\alpha,p)}(G^S)$ *are obtained for* G^S *and we derive some results for the partial weight dominating vertex sets of a S-valued graphs. Also we establish a relationship between two partial weight domination numbers of S-valued graph* $\gamma_{(\alpha,p)}(G^S)$ *and* $\gamma_{(\alpha,q)}(G^S)$*where* $0 \leq p < q \leq 1$*,which in turn provides another upper bound for*$\gamma_{(\alpha,p)}(G^S)$*. Further, we focus special attention on (α, 1/2) domination number of S-valued graphs.*

1. INTRODUCTION:

More than 4000 papers were published on dominating sets in graphs and in all that papers, properties of variety of variations of dominating sets and good bounds for various domination numbers were derived. Many variations of the dominating sets can be found in graphs, most of which are motivated by many real life situations.

We consider one such real life situation in the following street lights problem. There are *n* street lights in the street and each light is focusing on particular length of the street. Each vertex represents the street lights and *u* and *v* are adjacent if and only if the two lights focus each other. In this case the most economical solution is the minimum number of possible lights that cover the streets, correspond to the γ- sets.Suppose that due to the repair of lights or the short circuit, we can at most secure a fraction or part of the street lights and keep switched off the lights that are needed to be repaired on that particular day. In that case, we need partial weight domination in S-valued graphs and the length is nothing but the weight of the graph. In this present work, we define partial domination parameter in S-valued graphs and derive some results for the $\gamma_{(\alpha,p)}$-sets, that is the partial weight domination sets of S-valued graphs and partial weight domination number of certain S-valued graphs.

2. PRELIMINARIES

Definition 2.1. [4] A Semiring (S, +,.) is an algebraic system with a non-empty set S together with + and • such that

1. (S, +, 0) is a monoid.
2. (S, •) is a semigroup.
3. For all a,b,c ∈ S, a•(b+c)=a•b+a•c and (a+b)•c=a•c+b•c
4. 0•x=x•0=0∀ x∈ S

Definition 2.2. [3] Let (S, +, •) be a semiring. A relation ≼ is said to be a canonical pre-order if for a, b ∈ S, a≼b if and only if there exists c∈S such that a+c =b

Definition 2.3. [3] Let G = (V, E⊂ VXV) be the underlying graph with both V, E≠ ∅. For any semiring (S, +, •) a semiring valued graph (or an S-valued graph) G^S is defined to be the graph G^S = (V, E,σ, ψ) where σ: V→S and ψ:E→S is defined to be

$$\psi(x,y) = \begin{cases} \min(\sigma(x),\sigma(y)), & if\ \sigma(x) \preccurlyeq \sigma(y)\ or\ \sigma(y) \preccurlyeq \sigma(x) \\ 0 & otherwise \end{cases}$$

For every unordered pair (x,y) of E⊂VXV we call σ a S-vertex set and ψ an S-edge set of S-valued graph G^S

Definition 2.4. [3] Consider the S-valued graph G^S = (V, E, σ, ψ). For v_i ∈V, the open neighborhood of v_i in G^S is defined as a subset of V x S such that that

$N_S(v_i)$= {$(v_i, \sigma(v_j))/(v_i, v_j)$ ∈E, $\psi(v_i, v_j)$) ∈S}. For $v_i \in V$ a closed neighborhood of v_i in G^S is defined to be the subset of VXS such that $N_S[v_i]$= $N_S(v_i)$ ∪{$(v_i, \sigma(v_i))$}

Definition 2.5. [3] The degree of the vertex v_i of the S-valued graph $G^S$is defined as

$deg_S(v_i)$=($\sum_{(v_i,v_j)\in E} \psi(v_i, v_j)), l$) where l is the number of edges incident with v_i

Definition 2.6. [3] In the S-valued graph G^S = (V, E,σ, ψ), to compare the degrees of two vertices $v, w \in G^S$, we define the ≼ as follows:

- ($\sigma(v)$,deg(v)) ≼($\sigma(w)$,deg(w))⇔ ($\sigma(v) \preccurlyeq \sigma(w)$) and deg($v$) ≤ deg($w$)
- If ($\sigma(v) \preccurlyeq \sigma(w)$) and deg($v$) ≥deg($w$), the comparison is with respect to the S-values

Definition 2.7. [3] Let G^S = (V, E,σ, ψ) be a given S-valued graph. A vertex v in G^S is said to be a weight dominating vertex if $\sigma(u) \preccurlyeq \sigma(v)\ \forall u \in N_S[v]$

Definition 2.8. [3] A subset D⊆V is called a weight dominating vertex set of G^Sif for each $v \in D$ $\sigma(u) \preccurlyeq \sigma(v)$ ∀ u∈ $N_S[v]$. The minimum cardinality of a weight dominating set of G^S is called a weight domination number of G^S which is denoted by $\gamma^S(G^S)$ and the corresponding weight dominating set is called a $\gamma^S -$ set of G^S.

Definition 2.9. [3] Let G^S= (V, E,σ, ψ) be a given S-valued graph. The cardinality of the minimal weight dominating vertex set D⊆V is called the weight dominating vertex number of G^S which is denoted by $\gamma^S(G^S)$ That is

$$\gamma^S(G^S) = \min\{(|D|s, |D|)/D \text{ is a weight dominating set vertex set of } G^S\}$$

Here $|D|s$ denotes the scalar cardinality of D and $|D|$ denotes the number of vertices in D

Definition 2.10. [3] Let G^S = (V, E,σ, ψ) be a given S-valued graph. If D is a weight dominating vertex set of G^S then the scalar cardinality of D denoted by $|D|s$ is defined by $|D|s = \sum_{v \in D} \sigma(v)$

Definition 2.11. [1] The complement $\bar{G}$ of a simple graph G is the simple graph with vertex set V, two vertices being adjacent in $\bar{G}$ iff they are not adjacent in G.

Definition 2.12. [1] A Dominating set D⊆V of a graph G is said to be a global dominating set if D is also a dominating set in the complement of G.

3. PARTIAL WEIGHT DOMINATING VERTEX SETS IN S-VALUED GRAPHS

Definition3.1. For any S – valued graph G^S and proportion p ε [0,1] with some αεS a set D⊆V is a (α,p) partial weight dominating vertex set if (α,p)≼($|N(D)|_S$, $|N(D)|/|V|$)

Definition3.2. The (α,p) partial weight domination number $\gamma_{(\alpha,p)}(G^S)$ is the minimum cardinality of a (α,p) partial weight dominating vertex set of $G^S$and it is given by $\gamma_{(\alpha,p)}(G^S) = min\{(|D|_S, |D|)\}$ where D is a (α,p) partial weight dominating vertex of G^S where $|D|_S$ denotes the scalar cardinality of D and $|D|$ denotes the number of vertices in D.

Here we note that a $\gamma_{(\alpha,p)}$ set is not in general related to a γ- set.In particular a γ- set does not necessarily contain a $\gamma_{(\alpha,p)}$ set.Equivalently a $\gamma_{(\alpha,p)}$ set can not necessarily be extended to γ-set.

Clearly (0,1)≼ $\gamma_{(\alpha,p)}(G^S)$ ≼ $\gamma(G^S)$ and $\gamma_{(\alpha,1)}(G^S) = \gamma(G^S)$

For example, we say that a set D⊆V is a ½- weight dominating vertex set if (α,1/2)≼($|N(D)|_S$, $|N(D)|/|V|$).The ½-weight domination number $\gamma_{(\alpha,1/2)}$ equals the minimum cardinality of a ½- weight dominating vertex set in G^S

Example 3.3.

consider the semiring (S={0,a,b,c},+, .} with the following Cayley tables.

+	0	a	b	c
0	0	a	b	c
a	A	a	b	c
b	B	b	b	b
c	C	c	b	b

•	0	a	b	c
0	0	0	0	0
a	0	0	a	0
b	0	a	b	c
c	0	0	c	c

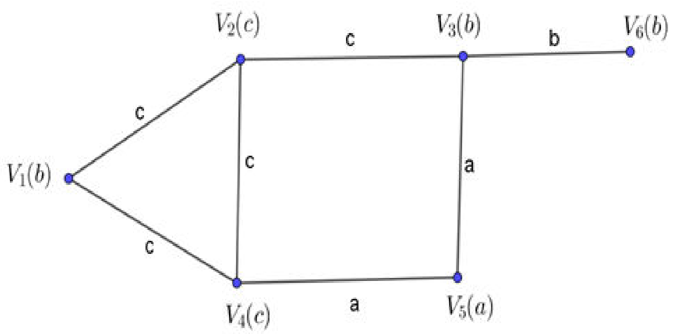
Let ≼ be a canonical preorder in S given by

$0 \preccurlyeq 0, 0 \preccurlyeq a, 0 \preccurlyeq b, 0 \preccurlyeq c, a \preccurlyeq a, a \preccurlyeq b, a \preccurlyeq c, b \preccurlyeq b, c \preccurlyeq c, c \preccurlyeq b$

Here σ: V→S and ψ:E→S are defined to be σ(v_1) = σ(v_6)= σ(v_3)=b σ(v_2)= σ(v_4)= c σ(v_5)=a and

Ψ(v_1v_2)= Ψ(v_2v_3)= Ψ(v_1v_4)= Ψ(v_2v_4)=c Ψ(v_1v_6)=a Ψ(v_3v_6)=b and Ψ(v_4v_5)= Ψ(v_3v_5)=c

Consider the S – valued graph G^S = (V, E,σ, ψ)



Let us fix α to be a

Consider the proportion p to be 1/2 then the (α,p) partial weight dominating vertex set is $D = \{(v_1, \sigma(v_1))\}$

N(D)= $\{(v_1, \sigma(v_1)), (v_2, \sigma(v_2))(v_6, \sigma(v_6))\}$

$|N(D)|_S = \sigma(v_1) + \sigma(v_2) + \sigma(v_6) = b + c + b = b$ and $|N(D)| = 3$

Here (α,p)≼($|N(D)|_S$, $|N(D)|/|V|$)

(a,1/2) ≼ (b,3/6)

≼(b,1/2)

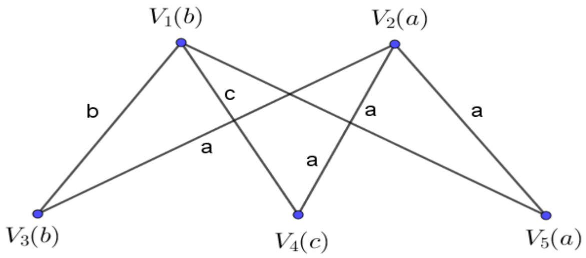
Here the (α,1/2) partial weight domination number $\gamma_{(\alpha,1/2)}(G^S) = (b, 1)$

Example3.3: Consider $K_{2,3}^S$ with the semiring mentioned in example 1

Let ≼ be a canonical preorder in S given by

0 ≼ 0, 0 ≼ a, 0 ≼ b, 0 ≼ c, a ≼ a, a ≼ b, a ≼ c, b ≼ b, c ≼ c , c ≼ b

Here σ: V→S and ψ:E→S are defined to be σ(v_1) = σ(v_3)=b σ(v_2)= σ(v_5)= a σ(v_4)=c and Ψ(v_2v_3)= Ψ(v_2v_4)= Ψ(v_2v_5)= Ψ(v_1v_5)=a Ψ(v_1v_3)=b and Ψ(v_1v_4)=c



Choose α=a and p to be ½

Consider D=$\{(v_1, \sigma(v_1))\}$

N(D)= $\{(v_1, \sigma(v_1)), (v_3, \sigma(v_3))(v_4, \sigma(v_4))(v_5, \sigma(v_5))\}$

$|N(D)|_S = \sigma(v_1) + \sigma(v_3) + \sigma(v_4) + \sigma(v_5) = b + a + c + b = b$ and $|N(D)| = 4$

Then (α,p)≼($|N(D)|_S$, $|N(D)|/|V|$)

(a,1/2)≼(b,4/5)

$\gamma_{(\alpha,p)}(K_{2,3}^S) = (\sum_{v \in V} \sigma(v), 1)$

4. RESULTS ON PARTIAL WEIGHT DOMINATION NUMBER

Proposition 4.1: Let G^S be a S-valued graph on n vertices then $\gamma_{(\alpha,p)}(G^S) = (\sum_{v \in V} \sigma(v), 1)$ for all $p \in (0, \frac{\Delta+1}{n}]$

Proposition 4.2: Let G^S be a S-valued graph on n vertices then $\gamma_{(\alpha,p)}(G^S) = \gamma(G^S)$ for all $p \in (1 - 1/n, 1]$

Proposition 4.3: Let C_n^S be the S-valued cycle on n vertices and P_n^S be the S-valued path on n vertices then $\gamma_{(\alpha,p)}(C_n^S) = \gamma_{(\alpha,p)}(P_n^S) = \left(\sum_{v \in V} \sigma(v), \left\lceil \frac{np}{3} \right\rceil\right)$

Proof: Let D be a (α,p) partial weight dominating vertex set of C_n^S then $|N(D)|_S$ ≼ $\sum_{v \in V} \sigma(v)$ and $\left\lceil \frac{np}{3} \right\rceil \leq |N(D)|$

To dominate $\lceil np \rceil$ vertices in C_n^S, we need at least $\left\lceil \frac{np}{3} \right\rceil$ vertices then

$\gamma_{(\alpha,p)}(C_n^S) = (\sum_{v\in V}\sigma(v), |D|)$ and $|D| = \left\lceil\frac{\lceil np\rceil}{3}\right\rceil = \left\lceil\frac{np}{3}\right\rceil$

Hence $\gamma_{(\alpha,p)}(C_n^S) = \left(\sum_{v\in V}\sigma(v), \left\lceil\frac{np}{3}\right\rceil\right)$

Similarly $\gamma_{(\alpha,p)}(P_n^S) = \left(\sum_{v\in V}\sigma(v), \left\lceil\frac{np}{3}\right\rceil\right)$

Proposition 4.4: For any S-valued complete graph vertices K_n^S , $\gamma_{(\alpha,p)}(K_n^S) = (\sum_{v\in V}\sigma(v), 1)$

Proposition 4.5: For any S – valued complete bipartite graph $K_{m,n}^S$ with 2≤m≤n,

$$\gamma_{(\alpha,p)}(K_{m,n}^S) = \begin{cases} (\sum_{v\in V}\sigma(v), 1) \ \ if\ 0 \le p \le \frac{m+1}{m+n} \\ (\sum_{v\in D}\sigma(v), 2), if\ p \le \frac{m+1}{m+n} \le 1 \end{cases}$$

Now we compare $\gamma_{(\alpha,p)}(G^S)$ and $\gamma_{(\alpha,q)}(G^S)$ for different proportions p and q.

Proportion 4.6: Let 0≤p<q≤1 for some α ε S then $\gamma_{(\alpha,p)}(G^S) \preccurlyeq \gamma_{(\alpha,q)}(G^S)$

Proof: We know that every (α,q) partial weight dominating vertex set of G^S is a (α,p) partial weight dominating vertex set of G^S.

More over equality holds if and only if the $\gamma_{(\alpha,p)}$ partial weight dominating vertex set dominates a proportion q of the vertices.

Setting q=1 we get $\gamma_{(\alpha,1)}(G^S) = \gamma(G^S)$

We get a relation between weight domination number and partial weight domination number.

Corollary4.6.1: The upper bound for partial weight domination number is given by $\gamma_{(\alpha,1)}(G^S) \preccurlyeq \gamma(G^S)$

Theorem4.7: Let G^S be a S-valued graph with weight domination number $\gamma(G^S)$ then for all p ε (0,1) $\gamma_{(\alpha,p)}(G^S) + \gamma_{(\alpha,1-p)}(G^S) \preccurlyeq \gamma(G^S) + (\sum_{v\in V}\sigma(v), 1)$

Proof: Let D be a $\gamma(G^S)$ set and p ε (0,1).Let D_1 be the subset of D with np≥$|N(D_1)|$ and $|N(D_1)|_S = \sum_{v\in V}\sigma(v)$ such that D_1 is a minimal subset of D with this property. Clearly $\gamma_{(\alpha,p)}(G^S) \preccurlyeq (\sum_{v\in V}\sigma(v), |D_1|)$.

Let $D_2 = D\backslash D_1$ and $(v,\sigma(v)) \in D_1$

Since D_1 is minimal with respect to the above property we have $\left|N\left(D_1\backslash(v,\sigma(v))\right)\right| < np$

Now, as D=$\left(D_1\backslash(v,\sigma(v))\right) \cup D_2 \cup \{(v,\sigma(v))\}$

$$\begin{aligned} n=|V| &= N(D) \\ &\le N\left[\left(D_1\backslash(v,\sigma(v))\right)\right] + N[D_2 \cup \{(v,\sigma(v))\}] \\ &< np + N[D_2 \cup \{(v,\sigma(v))\}] \\ N[D_2 &\cup \{(v,\sigma(v))\}] > n(1-p) \end{aligned}$$

Thus $D_2 \cup \{(v,\sigma(v))\}$ is an $(1-p)$ partial weight dominating vertex set of G^S and $\gamma_{(\alpha,1-p)}(G^S) = (|D_2 \cup \{(v,\sigma(v))\}|_S, |D_2 \cup \{(v,\sigma(v))\}|)$

$$\gamma_{(\alpha,1-p)}(G^S) \preccurlyeq ((\sum_{v\in V}\sigma(v), |D_2| + 1),$$

$$\begin{aligned} \gamma_{(\alpha,p)}(G^S) + \gamma_{(\alpha,1-p)}(G^S) &\preccurlyeq ((\sum_{v\in V}\sigma(v), |D_1| + |D_2| + 1), \\ &\preccurlyeq ((\sum_{v\in V}\sigma(v), |D| + 1) \\ &\preccurlyeq \gamma(G^S) + (\sum_{v\in V}\sigma(v), 1) \end{aligned}$$

Theorem 4.8: Let G^S be a S-valued graph with weight domination number $\gamma(G^S)$.For any positive integer k≥2 with $p_1 + p_2 + \cdots + p_k \leq 1$and $p_i \in (0,1)$ for all i, $\gamma_{(\alpha,p_1)}(G^S) + \gamma_{(\alpha,p_2)}(G^S) + \cdots + \gamma_{(\alpha,p_k)}(G^S) \preccurlyeq \gamma(G^S) + (\sum_{v\in V} \sigma(v), k/2)$

Proof: We prove it by induction on k.

For k=2, $p_1 + p_2 \leq 1$,hence by above theorem,

$\gamma_{(\alpha,p_1)}(G^S) + \gamma_{(\alpha,p_2)}(G^S) \preccurlyeq \gamma_{(\alpha,p)}(G^S) + \gamma_{(\alpha,1-p)}(G^S) \preccurlyeq \gamma(G^S) + (\sum_{v\in V} \sigma(v), 1)$

Assume that k>2 and the theorem holds for integers less than k. Then at least one value of p_i must satisfy $p_i \leq 1/2$.Without loss of generality, let $p_k \leq 1/2$ By Corollary 4.1

$\gamma_{(\alpha,1/2)}(G^S) \preccurlyeq \gamma(G^S) + (\sum_{v\in V} \sigma(v), \lceil 1/2 \rceil)$

Finally using the induction hypothesis, we get

$$[\gamma_{(\alpha,p_1)}(G^S) + \gamma_{(\alpha,p_2)}(G^S) + \cdots + \gamma_{(\alpha,p_{k-1})}(G^S)] + \gamma_{(\alpha,p_k)}(G^S)$$
$$\preccurlyeq \gamma(G^S) + (\sum_{v\in V} \sigma(v), \frac{k-1}{2}) + \gamma(G^S) + (\sum_{v\in V} \sigma(v), \lceil 1/2 \rceil)$$
$$\preccurlyeq \gamma(G^S) + (\sum_{v\in V} \sigma(v), k/2)$$

Hence proved the theorem.

Theorem4.9: Let G^S be a S-valued graph with components G_1^S, G_2^S,….. G_k^S Then $\gamma_{(\alpha,p)}(G^S) \preccurlyeq \sum_{i=1}^{k} \gamma_{(\alpha,p)}(G_i^S)$

Proof: Let D_i be a $\gamma_{(\alpha,p)}$ set of G_i^S , for i=1,2,….k.Then $p|V(G)| \preccurlyeq |N(D_i)|$ for i=1,2,….k. Let D=$D_1 \cup D_2 \cup \ldots \cup D_k$.Thus $|N(D)| = \sum_{i=1}^{k} |N(D_i)|$

p$\sum_{i=1}^{k} |V(G_i^S)| \preccurlyeq \sum_{i=1}^{k} |N(D_i)|$ and therefore $p|V(G)| \preccurlyeq |N(D)|$ and D is a $\gamma_{(\alpha,p)}$ set of G^S and hence $\gamma_{(\alpha,p)}(G^S) \preccurlyeq \sum_{i=1}^{k} \gamma_{(\alpha,p)}(G_i^S)$

Theorem4.10: For any connected S-valued graph G^S $\gamma_{(\alpha,i/j)}(G^S) \preccurlyeq \gamma(G^S) + (\sum_{v\in V} \sigma(v), \lceil i/j \rceil)$

Proof: Given a γ- set D=$\{(v_1, \sigma(v_1)), (v_2, \sigma(v_2)) \ldots \ldots .. (v_r, \sigma(v_r))\}$ partition V into sets $D_1, D_2, \ldots \ldots D_r$ such that$D_i \subseteq N[v_i]$, $v_i \in D_i$

Without loss of generality,$|D_1| \geq \cdots \ldots \ldots \ldots . \geq |D_r|$.

Define D'=$\{(v_1, \sigma(v_1)), (v_2, \sigma(v_2)) \ldots \ldots .. (v_{\lceil ir/j \rceil}, \sigma(v_{\lceil ir/j \rceil}))\}$

Claim:$\left| \bigcup_{k=1}^{\lceil ir/j \rceil} D_k \right| \geq i/j|V|$

By construction $\left| \bigcup_{k=1}^{\lceil ir/j \rceil} D_k \right| + \left| \bigcup_{k=\lceil ir/j \rceil+1}^{r} D_k \right| = |V|$

Since the average size of D_k, k=1,2…..$\lceil ir/j \rceil$ is atleast the average size of all D_k's,the result become true because at worst $|D_k| = |D_l|$ for all k≠l and here $\left| \bigcup_{k=1}^{\lceil ir/j \rceil} D_k \right| \geq i/j|V|$

Corollary 4.10.1: For any connected S-valued graph G^S $\gamma_{(\alpha,1/2)}(G^S) \preccurlyeq \gamma(G^S) + (\sum_{v\in V} \sigma(v), \lceil 1/2 \rceil)$

Next consider some Nordhaus-Gaddum type bounds on the i/j-partial weight domination number on S-valued graphs

Theorem 4.11: If $G^S$and $\bar{G}^S$ are connected S-valued graphs then $\gamma_{(\alpha,i/j)}(G^S) + \gamma_{(\alpha,i/j)}(\bar{G}^S) \preccurlyeq (\sum_{v\in V} \sigma(v), n + 2\lceil i/j \rceil)$

Proof: Applying theorem for both $G^S$and $\bar{G}^S$ we get that

$\gamma_{(\alpha,i/j)}(G^S) \preccurlyeq \gamma(G^S) + (\sum_{v\in V} \sigma(v), \lceil i/j \rceil)$

$\gamma_{(\alpha,i/j)}(\bar{G}^S) \preccurlyeq \gamma(\bar{G}^S) + (\sum_{v\in V}\sigma(v), \lceil i/j \rceil)$

Adding these two gives

$\gamma_{(\alpha,i/j)}(G^S) + \gamma_{(\alpha,i/j)}(\bar{G}^S) \preccurlyeq \gamma(G^S) + (\sum_{v\in V}\sigma(v), \lceil i/j \rceil) + \gamma(\bar{G}^S) + (\sum_{v\in V}\sigma(v), \lceil i/j \rceil)$

$\preccurlyeq \gamma(G^S) + \gamma(\bar{G}^S) + (\sum_{v\in V}\sigma(v), 2\lceil i/j \rceil)$

$\preccurlyeq (\sum_{v\in V}\sigma(v), n) + (\sum_{v\in V}\sigma(v), 2\lceil i/j \rceil)$ since we already had if G^S has no S-isolates then $\gamma(G^S) + \gamma(\bar{G}^S) \preccurlyeq (\sum_{v\in V}\sigma(v), n)$ where n is the number of vertices of G^S

$\preccurlyeq (\sum_{v\in V}\sigma(v), n + 2\lceil i/j \rceil)$

Corollary 4.11.1: If G^S and $\bar{G}^S$ are connected S-valued graphs then

$\gamma_{(\alpha,1/2)}(G^S) + \gamma_{(\alpha,1/2)}(\bar{G}^S) \preccurlyeq (\sum_{v\in V}\sigma(v), n + 2)$

5. CONCLUSIONS:

In S-valued graphs, we derived some results for partial weight dominating vertex sets and partial weight domination number. Further we have to give the generalization result for the upper bound of this (α, p)-partial weight domination for G^S.

REFERENCES:

[1] Bondy J Murty, 1984, *Graph Theory with applications*, U.S.R. (Elsevier Science Publishing Co, Sixth printing).

[2] Haynes T W, Hedetniemi S T and Slater P J, 1998, *Fundamentals of Dominations in Graphs*, New York (Monographs and Textbooks in Pure and Applied Mathematics, Marcel Dekker Inc.,).

[3] S Jeyalakshmi and M Chandramouleeswaran, Sep-Oct2016, *Vertex Domination on S-valued graphs*, IOSR Journal of Mathematics (IOSR-JM), e-ISSN: 2278-5728, p-ISSN: 2319-765X, Volume 12, Issue 5 Ver. IV, PP 08-12.

[4] Jonathan Golan, *Semiring and their Applications*, London (Kluwer Academic Publishers).

[5] Rajkumar M, Jeyalakshmi S and Chandramouleeswaran M, 2015, *Semiring valued graphs*, International Journal of Math.Sci.and Engg.Appls.,9(3), PP 141-152

[6] J.E.Dunbar,D.G.Hoffman,R.C.Lasker,andL.RMarkus,α-domination,Disctete Mathematics,Vol.211,11-26,2000.

[7] Benjamin.M.Case,Stephen T. Hedetniemi,Renu C.Laskar and Drew J.Lipman Partial domination in graphs,arXiv:1705.03096v1[math.CO] 8 May 2017.

[8] Angsuman Das,Partial Dominaion in Graphs,arXiv:1707.04898v2[math.CO] 24July 2017.

AN ANALYTIC APPROACH ON THE WITHIN-HOST MATHEMATICAL MODEL OF COVID-19

S. Pavithra[1*], S. Dharini[2]

[1*] Assisstant Professor, PG & Research Department of Mathematics,
The Standard Fireworks Rajarathnam College for Women, Sivakasi-626123, India.

[2] Research Scholar, PG & Research Department of Mathematics,
The Standard Fireworks Rajarathnam College for Women, Sivakasi-626123, India.
[*]Corresponding Author mail address: pavithra-mat@sfrcollege.edu.in

ABSTRACT: *A within-host mathematical model on the inflammatory mediators in COVID-19 is presented. Homotopy Perturbation Method (HPM) is discussed which is used to compute an approximate analytical expression for the concentrations of healthy type II Pneumocytes, infected type II Pneumocytes and viral load. The validity of HPM is analyzed using the function pde4, a function used to solve boundary value problems in MATLAB software. Graphical results confirm that (HPM) is in good agreement with the numerical solution adding to the accuracy and efficiency of (HPM) in finding the solution of the proposed model. The achieved results are applicable to the entire domain.*
Keywords: *Mathematical Modeling, COVID-19, Nonlinear initial value problem, Homotopy Perturbation Method.*

1. INTRODUCTION:

The outbreak of novel coronavirus in Wuhan, China marked the introduction of a virulent coronavirus into human society. The causative agent of this disease is identified as Severe Acute Respiratory Syndrome coronavirus-2 (SARS-CoV-2). The transmission of SARS-CoV-2 from a person to another occurs either through droplet infection or by a direct contact with an infected host. Also, transmissions from asymptotic carriers have also been reported. In spite of several researches being carried around the world, we are still lacking effective treatment approaches and epidemiological control measures. So, in order to break the natural history of the disease, it is inevitable to identify the possible interventions that help in reducing the severity of the virus and the growth of infected cells. Therefore, it is crucial to determine the coaction of viral growth along with the host immune response in the form of inflammatory mediators. In this paper, an analytical expression is derived for the ratio of healthy type II Pneumocytes $S(t)$,infected type II Pneumocytes I(t),viral load V(t) against time t by applying the method of Homotopy Perturbation. These analytical expressions can be useful in predicting the course of the disease over time and the simulation of novel therapies under various mechanisms.

2. MATHEMATICAL FORMULATION OF THE PROBLEM :

Recently *D.K.K. Vamsi et al.* [1] formulated a mathematical model with reference to the pathogens that deals with the natural history of covid-19. This is a first of its kind. Up to our knowledge there is no analytical solution for this system of nonlinear equations. The model is denoted as

$$\frac{dS}{dt} = \omega - \mu S - \beta SV \qquad (1)$$

$$\frac{dI}{dt} = \beta SV - (d_1 + d_2 + d_3 + d_4 + d_5 + d_6)I - \mu I$$

$$\frac{dI}{dt} = \beta SV - DI - \mu I \qquad (2)$$

$$\text{where } D = d_1 + d_2 + d_3 + d_4 + d_5 + d_6$$

$$\frac{dV}{dt} = \alpha I - (b_1 + b_2 + b_3 + b_4 + b_5 + b_6)V - \mu_1 V$$

$$\frac{dV}{dt} = \alpha I - BV - \mu_1 V \qquad (3)$$

$$\text{where } B = b_1 + b_2 + b_3 + b_4 + b_5 + b_6$$

where $S(t)$ represents the healthy type II Pneumocytes, $I(t)$represent the infected type II Pneumocytes , and $V(t)$ represent the viral load . Let ω be the natural birth rate of type II Pneumocytes. Let the natural birth rate of the virus $V(t)$ be α and the natural death rate be μ_1 .We suppose that infected type II Pneumocytes $I(t)$secrete virus $V(t)$that attacks the healthy type II Pneumocytes S(t) at rate β and the natural death rate of type II Pneumocytes be μ . With the release of cytokines and chemokines IL-6 TNF-a, INF-a, CCL5, CXCL8, CXCL10, the infected Pneumocytes and the virus are removed at the rate B and Ddie at rate μ_1 respectively. The parameters ω , β , μ , α , μ_1, B, D are positive constants . The initial conditions for the above equations as t=0 are $S = S_i$, $I = I_i$, $V = V_i$.

Table 1

Nomenclature

Parameters	Biological meaning
S	Healthy type II Pneumocytes
I	Infected Type II Pneumocytes
ω	Natural birth rate of Type II Pneumocytes
V	Viral load
β	Rate at which healthy Pneumocytes are infected
α	Burst rate of virus particles(rate at which infected cells release the virus particles)
μ	Natural death rate of Type II Pneumocytes
μ_1	Natural death rate of virus
$d_1, d_2, d_3, d_4, d_5, d_6$	Rates at which Infected Pneumocytes are removed because the release of cytokines and chemokines IL-6 TNF-α , INF-α , CCL5, CXCL8 , CXCL10 respectively
$b_1, b_2, b_3, b_4, b_5, b_6$	Rates at which Virus is removed because of the release of cytokines and chemokines IL-6 TNF-α , INF-α , CCL5, CXCL8 , CXCL10 respectively

3. ANALYTICAL SOLUTION FOR THE WITHIN-HOST MATHEMATICAL MODEL ON THE INFLAMMATORY MEDIATORS

Homotopy Perturbation method is a combination of topology and classic perturbation techniques. It is implemented to compute an approximate solution to a system of nonlinear differential equations pertaining to the problem. The efficiency of the Homotopy perturbation method for handling and solving various non-linear structures problems can be found in [2-5]. Ji Huan He employed the Homotopy perturbation method to solve the Lighthill equation [6], the Duffing equation [7] and the Blasius equation [8]. The homotopy perturbation method makes use of a small imbedding parameter p due to which very few iterations are required to achieve accurate result. The procedure for solving the non-linear differential equations, eqn. (1) - eqn. (3), by employing the method of homotopy perturbation is illustrated in Appendix A. The obtained results are as follows

$$S(t) = \frac{\omega}{\mu} + e^{-\mu t}(S_i - \frac{\omega}{\mu}) + \frac{\beta V_i \omega}{\mu B}(e^{-t(\mu+B)} - 1) + \frac{\beta V_i}{\mu + B}(e^{-t(2\mu+B)} - 1)(S_i - \frac{\omega}{\mu}) \quad (4)$$

$$I(t) = I_i e^{-t(D+\mu)} + \frac{\beta S_i V_i}{\mu + B - D}(1 - e^{-t(2\mu+B)}) \quad (5)$$

$$V(t) = V_i e^{-t(B+\mu)} + \frac{\alpha I_i}{D - B}(1 - e^{-t(D-B)})$$

(6)

where $S(t)$represents the healthy type II Pneumocytes, I(t) represent the infected type II Pneumocytes , and $V(t)$represent the viral load.

4. NUMERICAL SIMULATION

By implementing the Homotopy Perturbation Method, the non-linear differential equations governing the model (1)-(3) for the predetermined initial condition are established. These equations are illustrated numerically by making use of Matlab pdex 4 .The obtained solutions in comparison with the analytical solutions admit a remarkable accuracy.

5. RESULT AND DISCUSSION

Fig. 1 illustrates the ratio of healthy type II Pneumocytes $S(t)$, infected type II Pneumocytes I(t),viral load V(t) against time t. Fig. 2-4 presents plot of the ratio of healthy type II Pneumocytes $S(t)$ against time t by varying parameters $R1, R2, R3$ respectively. From Fig 2, it can be noted that the ratio of healthy type II Pneumocytes $S(t)$ increases steadily due to the increase in natural birth rate of type II Pneumocytes. From Fig. 3, it can be seen that there is an deterioration in the ratio of healthy type II Pneumocytes $S(t)$. This is due to the increase in rate at which healthy Pneumocytes are infected. Fig. 4 depicts that there is an decline in the ratio of healthy type II Pneumocytes $S(t)$ which is a consequence of the natural death rate of these cells. . Fig. 5-6 presents plot of the ratio of infected type II Pneumocytes $I(t)$ against time t by varying parameters $R3, R4$ respectively. From Fig. 5, it can be observed that the ratio of infected type II Pneumocytes $I(t)$ decreases steadily due to the increase in natural death rate of type II Pneumocytes. From Fig. 6, it can be noted that when the infected type II Pneumocytes are removed from the host the ratio of infected type II Pneumocytes $I(t)$ decreases. From this it can be inferred that the immunization drugs play a pivot role in stopping the spread of the infected cells. Fig. 7- 9 represents plot of the ratio of viral load $V(t)$ against time t by varying parameters $R5, R6, R7$ respectively. From Fig. 7, it can be noted that when the rate of removal of the virus from the host is high the ratio of the viral load decreases. From Fig. 8, it can be seen that ratio of the viral load $V(t)$decreases as the death rate of the virus increases. From Fig. 9, it can be observed that ratio of the viral load $V(t)$ increases when the rate of release of the virus from the infected cells is maximum. The higher the infected cells, the higher the viral load. Therapeutic agents which acts to improve the response of the host immune system in reducing the number of infected cells and viraload can be administered.

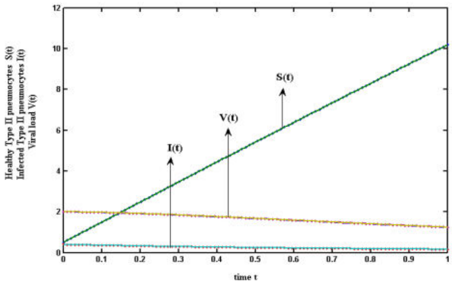


Fig. 1. Plot of healthy type II Pneumocytes $S(t)$ versus time. The correlation is assessed for Eq. (1-3) with the allotted values of the parameters $R1 = 10;\ R2 = 0.005;\ R3 = 0.05;\ R4 = 1.0238;\ R5 = 0.6240;\ R6 = 1.1;\ R7 = 8.2$.

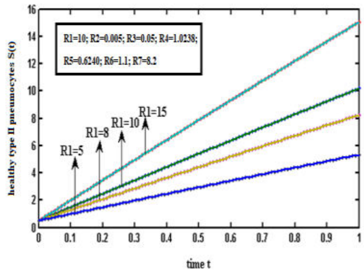


Fig. 2. Plot of healthy type II Pneumocytes $S(t)$ versus time t. The correlation is assessed for Eq. (1) with distinct values of the parameter R1 and the allotted values of other parameters $R2 = 0.005;\ R3 = 0.05;\ R4 = 1.0238;\ R5 = 0.6240;\ R6 = 1.1; R7 = 8.2$.

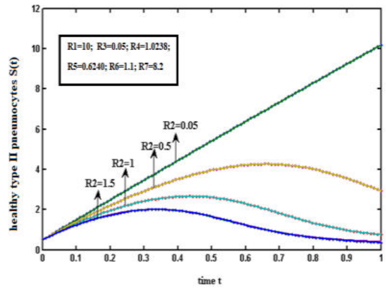


Fig. 3. Plot of healthy type II Pneumocytes $S(t)$ versus time t.The correlation is assessed for Eq. (1) with distinct values of the parameter R2 and the allotted values of other parameters $R1 = 10; R3 = 0.05;\ R4 = 1.0238;\ R5 = 0.6240;\ R6 = 1.1;\ R7 = 8.2$.

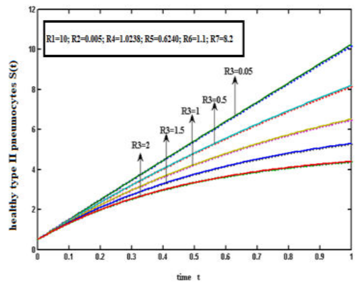


Fig. 4. Plot of healthy type II pneumoctes $S(t)$ versus time . The correlation is assessed for Eq. (1) with distinct values of the parameter R3 and the allotted values of other parameters $R1 = 10;\ R2 = 0.005; R4 = 1.0238;\ R5 = 0.6240;\ R6 = 1.1;\ R7 = 8.2$.

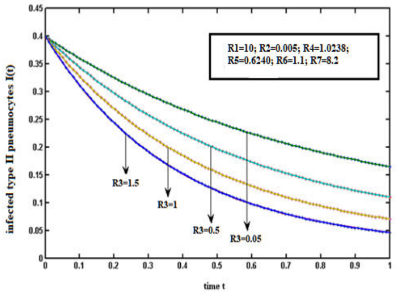


Fig. 5. Plot of infected type II pneumoctes $S(t)$ versus time t . The correlation is assessed for Eq. (2) with distinct values of the parameter R3 and the allotted values of other parameters $R1 = 10;\ R2 = 0.005;$ $R4 = 1.0238;\ R5 = 0.6240;\ R6 = 1.1;\ R7 = 8.2.$

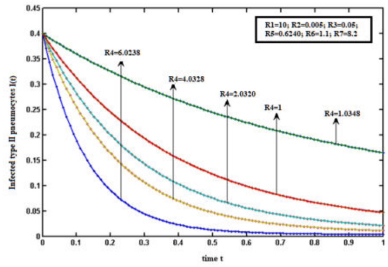


Fig. 6. Plot of infected type II pneumoctes $S(t)$ versus time t . The correlation is assessed for Eq. (2) with distinct values of the parameter R4 and the allotted values of other parameters $R1 = 10;\ R2 = 0.005;$ $R3 = 0{,}05;\ R5 = 0.6240;\ R6 = 1.1;\ R7 = 8.2.$

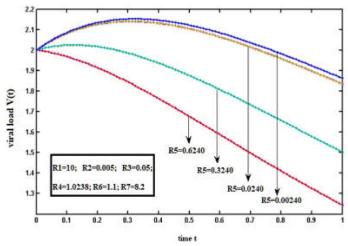


Fig. 7. Plot of viral load $V(t)$ versus time. The correlation is assessed for Eq. (3) with distinct values of the parameter R3 and the allotted values of other parameters $R1 = 10;\ R2 = 0.005; R3 = 0.05;\ R4 = 1.0238;\ R6 = 1.1; R7 = 8.2.$

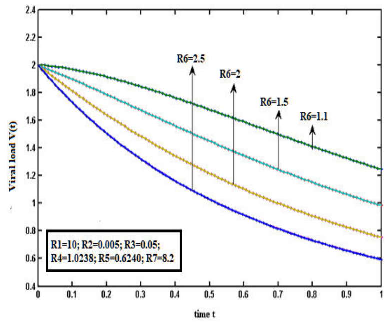


Fig. 8. Plot of viral load $V(t)$ versus time. The correlation is assessed for Eq. (3) with distinct values of the parameter R6 and the allotted values of other parameters$R1 = 10;\ R2 = 0.005; R3 = 0.05;\ R4 = 1.0238;\ R5 = 0.6240; R7 = 8.2.$

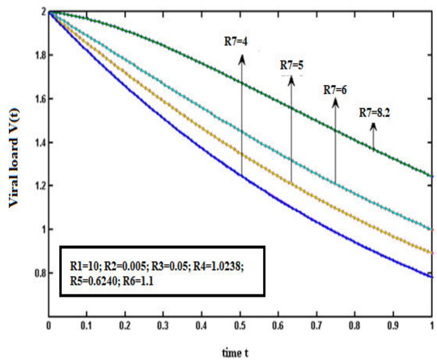


Fig. 9. Plot of viral load $V(t)$ versus time. The correlation is assessed for Eq. (3) with distinct values of the parameter R7 and the allotted values of other parameters $R1 = 10;\ R2 = 0.005; R3 = 0.05;$ $R4 = 1.0238;\ R5 = 0.6240;\ R6 = 1.1$

6. EXISTENCE AND UNIQUENESS OF THE SOLUTION :

Theorem 1 (Uniqueness of solution)

Let D denote the domain:

$$|t-t_0| \le a, \|x-x_0\| \le b, x=(x_1,x_2,...x_n), x_0 == (x_{10},x_{20},...x_{n0}) \qquad (6.1)$$

And suppose that *f(t,x)* satisfies the lipschitz condition: $\|f(t,x_1)-f(t,x_2)\| \le k\|x_1-x_2\|$, (6.2)

And whenever the pair *(t,x₁)* and *(t,x₂)* belong to domain D, where k is used to represent a positive constant.

Then, there exist a constant δ >0such that there exist a unique (exactly One)continuous vector solution x(t) of the system $x' = f(t,x), x(t_0) = x_0$ in the interval $\|t-t_0\| \le \delta$.

It is important to note that the condition (6.2) is satisfied by requirement that :

$$\frac{\partial f_i}{\partial x_j}, \qquad i,j=1,2,...,n \text{ be continuous and bounded in the domain D}$$

Lemma 1:

If *f(t, x)* has continuous partial derivative $\frac{\partial f_i}{\partial x_j}$, on a bounded closed convex domain ***R*** (i.e, convex set of real numbers), where ***R*** is used to denotes real numbers , then it satisfies a Lipschitz condition in ***R***. Our interest is in the domain:

$$1 \le \in \le \text{R}. \qquad (6.3)$$

So, we look for a bounded solution of the form $0 < \boldsymbol{R} < \infty$

Theorem 2:

Let D denote the domain defined in (6.1) such that (6.2) and (6.3) hold. Then there exist a solution of model system of equations (1)-(4) which is bounded in the domain D.

Proof:

Let $f_1 = \omega - \mu S - \beta SV \qquad (6.4)$

$$f_2 = \beta \text{SV} - \text{DI} - \mu \text{I} \qquad (6.5)$$

$$f_3 = \alpha \text{I} - \text{BV} - \mu_1 \text{V} \qquad (6.6)$$

We prove that $\frac{\partial f_i}{\partial x_j}, \quad i,j=1,2,...,n$ is continuous and bounded, Then the partial derivative of all the model equations are as follows.

From equation (6.4), $\frac{\partial f_1}{\partial S} = -\mu - \beta V, \left|\frac{\partial f_1}{\partial S}\right| = |-\mu - \beta V| < \infty, \quad \frac{\partial f_1}{\partial I} = 0, \left|\frac{\partial f_1}{\partial I}\right| = 0 < \infty,$

$$\frac{\partial f_1}{\partial V} = -\beta S, \left|\frac{\partial f_1}{\partial V}\right| = |-\beta S| < \infty,$$

From equation (6.5), $\frac{\partial f_2}{\partial S} = \beta V, \left|\frac{\partial f_2}{\partial S}\right| = |\beta V| < \infty, \quad \frac{\partial f_2}{\partial I} = -D - \mu, \left|\frac{\partial f_2}{\partial I}\right| = |-D - \mu| < \infty,$

$$\frac{\partial f_2}{\partial V} = \beta S, \left|\frac{\partial f_2}{\partial V}\right| = |\beta S| < \infty,$$

From equation (6.6), $\frac{\partial f_3}{\partial S}=0\left|\frac{\partial f_3}{\partial S}\right|=|0|<\infty,\qquad \frac{\partial f_3}{\partial I}=\alpha,\left|\frac{\partial f_3}{\partial I}\right|=|\alpha|<\infty,$

$$\frac{\partial f_3}{\partial V}=-\beta-\mu_1,\left|\frac{\partial f_3}{\partial V}\right|=|-\beta-\mu_1|<\infty,$$

Since all these partial derivatives are continuous and bounded, by Theorem (1), we can say that there exist a unique solution of (1)-(4) in the region D.

7. CONCLUSION :

In this paper, HPM is employed to attempt the solution of the model. Numerical simulations were performed to compare the analytical results obtained by HPM with numerical results. The results of the simulations were illustrated graphically. The results show that the analytical solution is in good agreement with the numerical results and produced accurately the same behavior. A clear conclusion can be drawn that HPM is highly reliable in finding the solution of a nonlinear differential Equation.

REFERENCES :

[1] D.K.K. Vamsi et al. "Within-host mathematical modeling on crucial inflammatory mediators and drug interventions in COVID-19 identifies combination therapy to be most effective and optimal," Alex. Eng. J. 60 (2021) 2491 – 2512.

[2] Q. K. Ghori, M. Ahmed, and A. M. Siddiqui, "Application of homotopy perturbation method to squeezing flow of a newtonian fluid" Int. J. Nonlin. Sci. Num. 8 (2007) 179–184.

[3] T. Öziş and A. Yildirim, "A comparative study of He's Homotopy Perturbation Method for determining frequency-amplitude relation of a nonlinear oscillator with discontinuities " Int. J. Nonlin. Sci. Num. 8 (2007) 243–248.

[4] S. J. Li and Y. X. Liu, "An improved approach to nonlinear dynamical system identifcation using PID neural networks," Int. J. Nonlin. Sci. Num. 7 (2006) 177–182.

[5] M. M. Mousa and S. F. Ragab, "Application of the homotopy perturbation method to linear and nonlinear schrödinger equations," Zeitschrift für Naturforschung A 63 (2008) 140–144.

[6]J. H. He, "Homotopy perturbation technique," Comput. Methods. Appl. Mech. Eng. 178 (199) 257–262.

[7] J. H. He, "Homotopy perturbation method: a new nonlinear analytical technique," Appl. Math. Comput. vol. 135 (2003) 73–79.

[8] J. H. He, "A simple perturbation approach to Blasius equation," Appl. Math. Comput. 140 (2003) 217–222.

DYNAMICAL ANALYSIS ON THE MATHEMATICAL MODEL OF A BIOREACTOR IN BATCH MODE WITH DECAY

R. Vanthana[1], S. Pavithra[2*] and K.P.V. Preethi[3]

[1] Research Scholar, School of Mathematics, Madurai Kamaraj University, Madurai 625021, Tamil Nadu, India.

[2*] Assistant Professor, PG and Research Department of Mathematics, The Standard Fireworks Rajaratnam College for Women, Sivakasi-626123, Tamil Nadu, India.

[3]Assistant Professor, Department of Mathematics, Saiva Bhanu Kshatriya College, Aruppukottai-626101, Tamil Nadu, India.
* Corresponding Author mail address: pavithra-mat@sfrcollege.edu.in

ABSTRACT: *Simple mathematical model for a bioreactor in batch mode with decay is presented. The two-dimensionaldifferential system describing the dynamics of thesubstrate andbiomass concentrations can be reduced to an algebraic equation for the biomass together with a single differential equation for the substrate from an analogy with the Henri Michaelis–Menten enzyme kinetic mechanism. The existence and uniqueness of the solution for the bioreactor model is discussed. The simple and closed form analytical expressions for the concentrations of biomass, and substrate have been derived by using New Homotopy Perturbation method for all values of parameter. Furthermore, in this work the numerical simulation of the problem is also reported using Matlab program to investigate the dynamics of the system. Graphical results are presented and discussed quantitatively to illustrate the solution. A satisfactory agreement between analytical and numerical results is noted.*

Keywords: *Mathematical Modeling, Bioreactor Model, New Homotopy Perturbation Method.*

1. INTRODUCTION:

A **bioreactor** may refer to any manufactured or engineered device or a system that supports a biologically active environmentin which living organisms and especially bacteria synthesize useful substances (as interferon) or break down harmful ones (as in sewage). This process can either be aerobic or anaerobic. They are commonly cylindrical, ranging in size from litres to cubic metres, and are often made of stainless steel.These devices are being developed for use in tissue engineering or biochemical engineering [1-4].

Application of bioreactor:

1. Producing biologic end-products (production bioreactor);
2. Cell or stem cell expansion (cell bioreactor); and
3. Tissue engineering (tissue bioreactor).

Mathematical formulation:

The two-dimensional differential system describing the dynamics of the substrate and biomass concentrations can be reduced to an algebraic equation for the biomass together with a single differential equation for the substrate. Then from an analogy with the Henri– Michaelis – Menten enzyme kinetic mechanism a simple model is proposed for a bioreactor in batch mode with decay [4].

2. TERMIMOLOGY AND DIFFERENTIAL EQUATIONS:

We investigate (in the spirit of [4]) some models of batch mode bioreactors with decay of the form:

$$\frac{ds}{dt} = -\alpha\mu(s)x \tag{1}$$

$$\frac{dx}{dt} = \mu(s)x - k_d x \tag{2}$$

Monod function [5]:

$$\mu_m(s) = \mu^* \frac{s}{K+s} \tag{3}$$

Substitute equation (3) in eqn. (1) & (2)

$$\frac{ds}{dt} = \frac{-\alpha\,\mu^* sx}{K+s} \tag{4}$$

$$\frac{dx}{dt} = \frac{\mu^* sx}{K+s} - k_d x \tag{5}$$

with positive initial conditions and positive α and k_d. Here s(t) is the concentrations of the substrate at time t, x(t) is concentration of biomass at time t, $\mu^* = \mu_{max}$ is the maximum specific substrate degradation rate, k_d is a decay (death rate) constant , μ(s) is a function depending on the substrate s, α the growth yield coefficient.
K are positive and represent different physical/biological quantities. With the initial conditions

At t=0, $s = s_i$ (6)

At t=0, $x = x_i$ (7)

Webb function [6]:

$$\mu_w(s) = \frac{\mu^* s(1+\frac{\beta s}{K_i})}{K+s+\frac{s^2}{K_i}} \tag{8}$$

Substitute equation (8) in eqn. (1) & (2)

$$\frac{ds}{dt} = \frac{-\alpha\,\mu^* s(1+\frac{\beta s}{K_i})x}{K + s + \frac{s^2}{K_i}} \tag{9}$$

$$\frac{dx}{dt} = \frac{\mu^* s(1+\frac{\beta s}{K_i})x}{K + s + \frac{s^2}{K_i}} - k_d x$$

(10)

With positive initial conditions and positive α and k_d. Here s(t) is the concentrations of the substrate at time t, x(t) is concentration of biomass at time t, μ^* does not represent the maximum of $\mu_w(s)$, k_d is a decay (death rate) constant ,μ(s) is a function depending on the substrate s,α the growth yield coefficient , K & K_i are positive and represent different physical/biological quantities , K_i is the inhibition constant, numerically equals the highest substrate concentration at which the specific growth rate is equal to one-half the maximum specific growth rate in the absence of inhibition, mass/volume. β is the Product formation constant. With the initial conditions

At t=0, $s = s_i$ (11)

At t=0, $x = x_i$ (12)

Nomenclature:

Symbol	Meaning	Numerical value
s	Concentration of substrate	1
x	Biomass	1
α	Growth yield coefficient	1.2
μ^*	Maximum specific substrate Degradation rate	3
k_d	Decay constant	1.4
K	Different biological quantity	2.3
β	Product formation constant	0.1
K_i	Inhibition constant	1

Uniqueness and Existence of Solution:

Lemma 3.1: Let $D \in R^n$ and $f : D \to R$ be a non-linear vector field. f is continuous and Lipchitz in $B = \{x \in D : \|x - x_o\| \le r\}$ for some real r with $r > 0$. Then, there exists some $\delta > 0$ such that $x' = f(t,x), x(t_0) = x_0$, has some unique solution.

Theorem 3.1: Suppose $F_i(t,x),\ x(t_0) = x_0, i = 1,2,3,4,5,6$ exists and unique in solution. Then the system satisfies Lipchitz condition.

Proof: Using the above lemma, it is enough if we prove that $\frac{\partial f_i}{\partial t_j}$ $i, j = 1,2,3,4$ is continuous and bounded in D.

Let

$$f_1 = \frac{-\alpha \mu^* sx}{K+s};\ f_2 = \frac{\mu^* sx}{K+s} - k_d x;\ f_3 = \frac{-\alpha \mu^* s(1+\frac{\beta s}{K_i})x}{K+s+\frac{s^2}{K_i}};\ f_4 = \frac{\mu^* s(1+\frac{\beta s}{K_i})x}{K+s+\frac{s^2}{K_i}} - k_d x$$

Now, we find the partial derivatives of these functions

For f_1,
$$\left|\frac{\partial f_1}{\partial s}\right| = \left|\frac{-\alpha \mu^* x}{K+s}\right| < \infty,\quad \left|\frac{\partial f_1}{\partial x}\right| = \left|\frac{-\alpha \mu^* s}{K+s}\right| < \infty$$

For f_2,
$$\left|\frac{\partial f_2}{\partial s}\right| = \left|\frac{-\alpha \mu^* x}{K+s}\right| < \infty\quad \left|\frac{\partial f_2}{\partial x}\right| = \left|\frac{-\alpha \mu^* s}{K+s} - k_d\right| < \infty$$

For f_3,
$$\left|\frac{\partial f_3}{\partial s}\right| = \left|\frac{-\alpha \mu^* (1+\frac{\beta s}{K_i})x}{K+s+\frac{s^2}{K_i}}\right| < \infty,\quad \left|\frac{\partial f_3}{\partial x}\right| = \left|\frac{-\alpha \mu^* (1+\frac{\beta s}{K_i})s}{K+s+\frac{s^2}{K_i}}\right| < \infty$$

For f_4,
$$\left|\frac{\partial f_4}{\partial s}\right| = \left|\frac{\mu^* (1+\frac{\beta s}{K_i})x}{K+s+\frac{s^2}{K_i}}\right| < \infty,\quad \left|\frac{\partial f4}{\partial x}\right| = \left|\frac{\mu^* s(1+\frac{\beta s}{K_i})}{K+s+\frac{s^2}{K_i}} - k_d\right| < \infty$$

Since the partial derivatives are continuous and bounded, we can conclude that the systems admits a unique solution.

Approximate Analytical Expression of the Concentration of substrate and biomass Using New Homotopy Perturbation Method (NHPM)

Presently, many authors have used the NHPM for solving various problems and have also exhibited its efficiency in solving the non-linear problems arising in the physics and engineering disciplines [7-10]. NHPM is the combination of topology and classical perturbation techniques. This has been used to solve non-linear boundary value problems, integral equations and many other problems [11]. Unlike other methods, NHPM uses only a few iterations to obtain an analytical expression and is very effective and simple. Using this method, we can obtain the following approximate solution for the concentration of substrate and biomass [4].

Numerical Simulation:

The non-linear differential eqns. (3.4)-(3.5), & (3.9)-(3.10) are solved using numerical methods. The function odex4 in Matlab software is used to solve this equation. The numerical solutions are then compared with the approximate analytical results.It can be inferred that the numerical results is in a good agreement with all the experimental values of the model parameters.

Monod function:

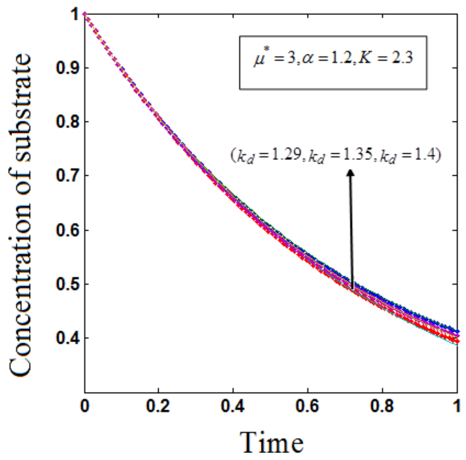


Figure 1: Plot of correlation between Numerical (dotted lines) and Analytical (solid lines) for the Concentration of Substrate s versus time t.

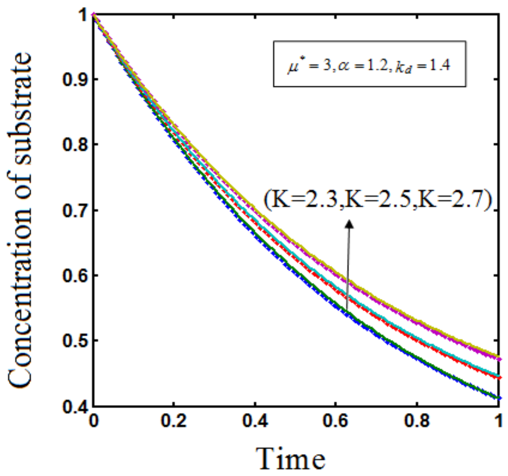


Figure 2: Plot of correlation between Numerical (dotted lines) and Analytical (solid lines) for the Concentration of Substrate s versus time t.

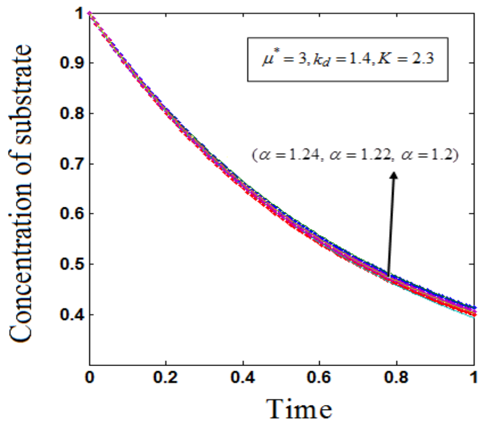


Figure 3: Plot of correlation between Numerical (dotted lines) and Analytical (solid lines) for the concentration of substrate s versus time t.

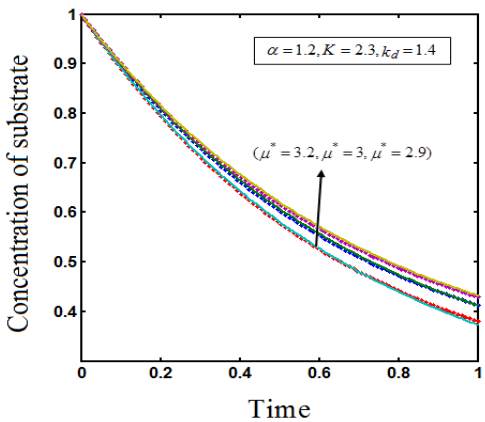


Figure 4: Plot of correlation between Numerical (dotted lines) and Analytical (solid lines) for the Concentration of Substrate s versus time t.

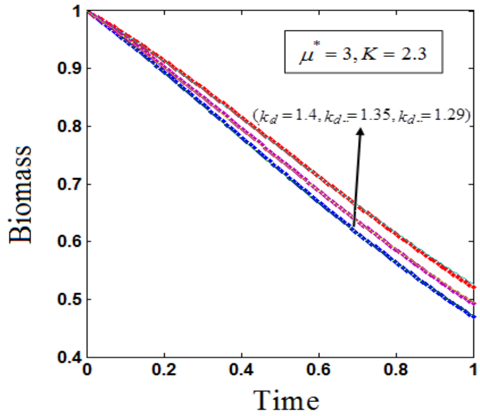


Figure 5: Plot of correlation between Numerical (dotted lines) and Analytical (solid lines) for the concentration of biomass x versus time t.

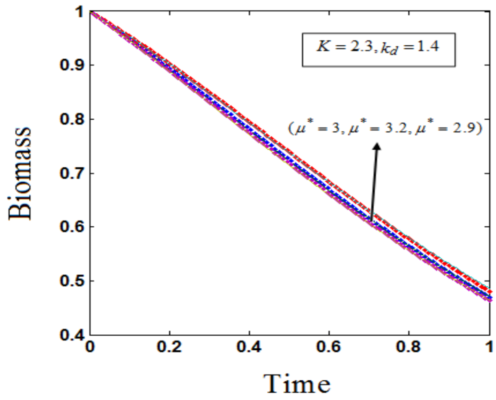


Figure 6: Plot of correlation between Numerical (dotted lines) and Analytical (solid lines) for the Concentration of Biomass x versus time t.

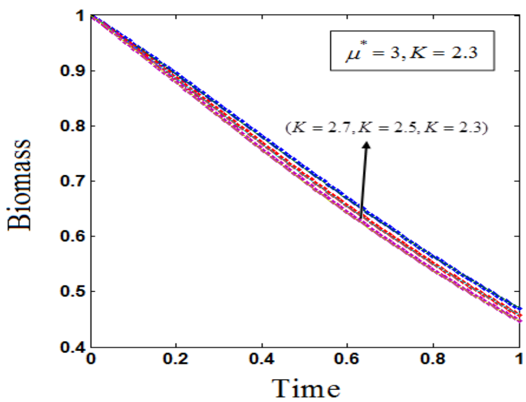


Figure 7: Plot of correlation between Numerical (dotted lines) and Analytical (solid lines) for the Concentration of Biomass x versus time t.

Webb function:

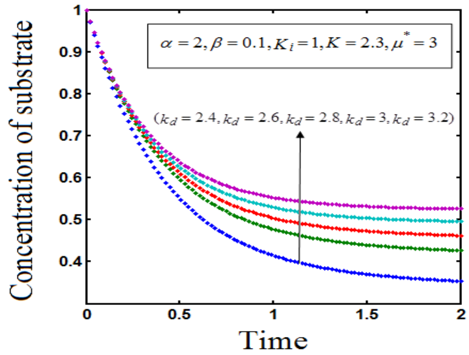


Figure 8: Plot of correlation between Numerical (dotted lines) and Analytical (solid lines) for the Concentration of Substrate s versus time t.

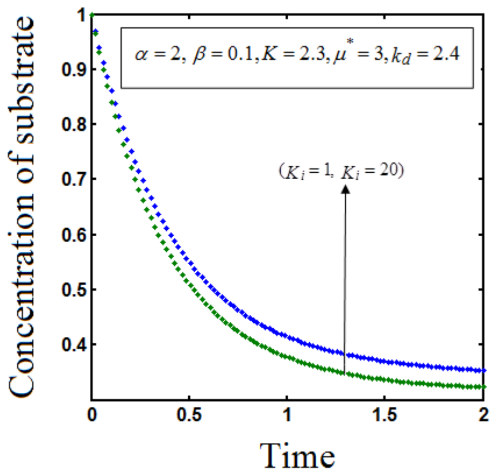


Figure 9: Plot of correlation between Numerical (dotted lines) and Analytical (solid lines) for the Concentration of Substrate s versus time t.

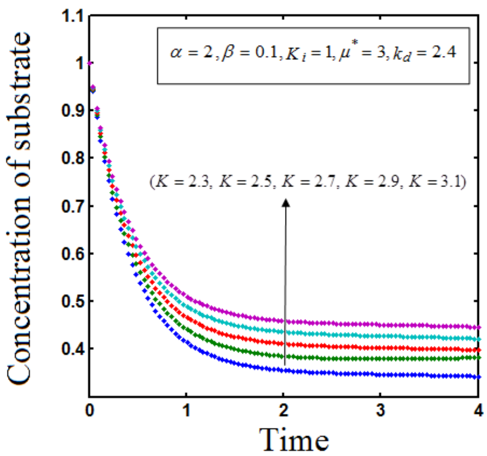


Figure 10: Plot of correlation between Numerical (dotted lines) and Analytical

(solid lines) for the Concentration of Substrate s versus time t.

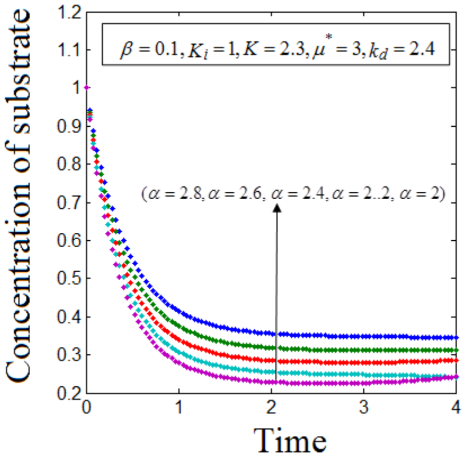


Figure 11: Plot of correlation between Numerical (dotted lines) and Analytical (solid lines) for the Concentration of Substrate s versus time t.

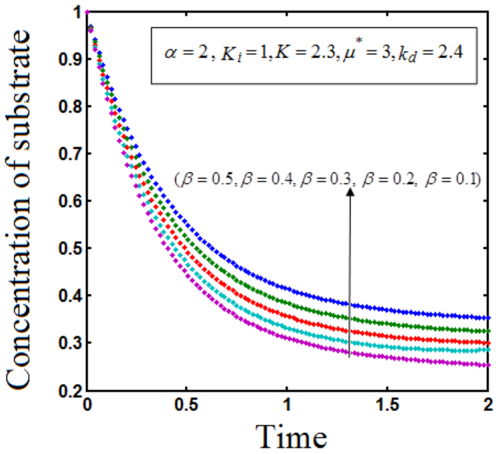


Figure 12: Plot of correlation between Numerical (dotted lines) and Analytical (solid lines) for the Concentration of Substrate s versus time t.

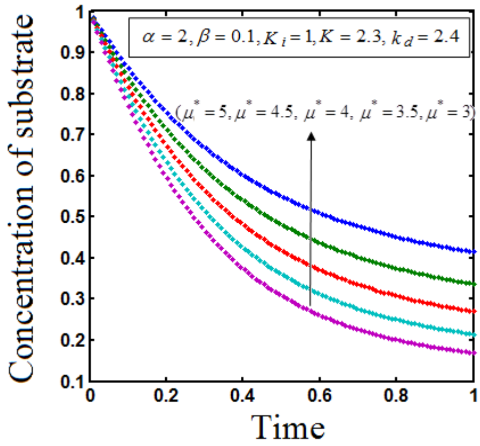


Figure 13: Plot of correlation between Numerical (dotted lines) and Analytical (solid lines) for the Concentration of Substrate s versus time t.

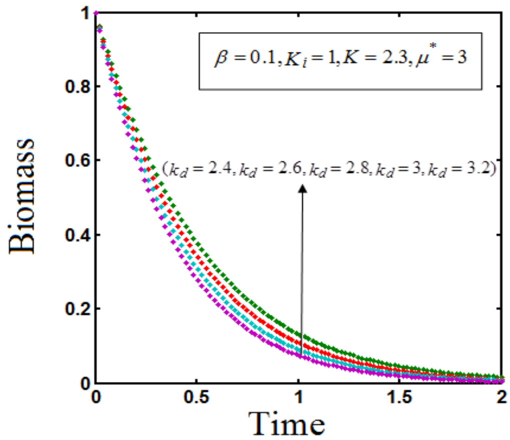


Figure 14: Plot of correlation between Numerical (dotted lines) and Analytical (solid lines) for the Concentration of Biomass x versus time t.

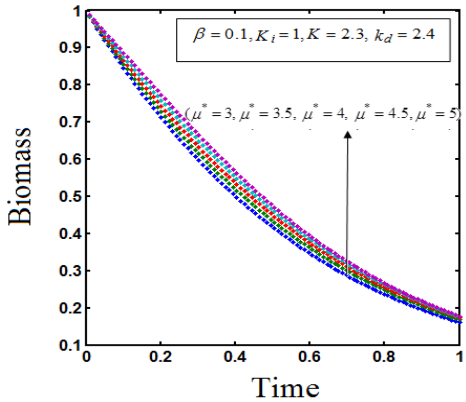


Figure 15: Plot of correlation between Numerical (dotted lines) and Analytical (solid lines) for the Concentrationof Biomass x versus time t.

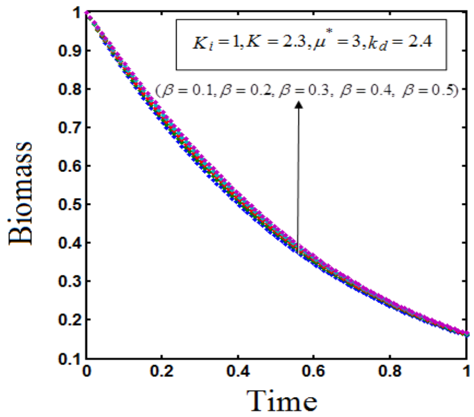


Figure 16: Plot of correlation between Numerical (dotted lines) and Analytical (solid lines) for the Concentration of Biomass x versus time t.

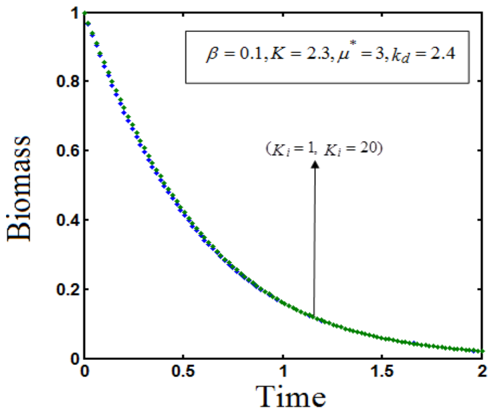


Figure 17: Plotof correlation between Numerical (dotted lines) and Analytical (solid lines) for the Concentrationof Biomass x versus time t.

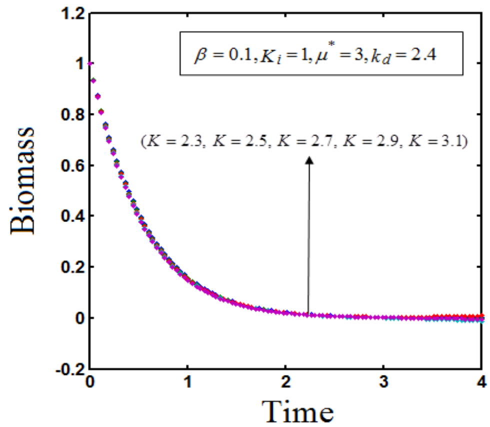


Figure 18: Plot of correlation between Numerical (dotted lines) and Analytical (solid lines) for the Concentration of Biomass x versus time t.

3. RESULT AND DISCUSSIONS:

The primary result of eqn. (3.23) - eqn. (3.24) represents the simple analytical expression pertaining to the Concentration of Substrate & Biomass respectively. Figure (1-4) represents the comparison of analytical and numerical stimulation of concentration of substrate verses time for different values of α, μ^*, k_d, K. From figure (1-2), it is inferred that concentration of substrate decreases when k_d, K increases for some fixed values of other parameter. From figure (3-4), it is inferred that concentration of substrate decreases when α, μ^* decreases for some fixed values of other parameter. Figure (5-7) represents the comparison of analytical and numerical stimulation of concentration of biomass verses time for different values of α, μ^*, k_d, K. From figure (5-6), it is inferred that concentration of biomass decreases when k_d, K decreases for some fixed values of other parameter. From figure (7), it is inferred that concentration of biomass decreases when μ^* increases for some fixed values of other parameter. The primary result of eqn. (3.35) - eqn. (3.36) represents the simple analytical expression pertaining to the Concentration of Substrate & Biomass respectively. Figure (8-13) represents the plot of concentration of substrate verses time for different values of $\alpha, \mu^*, k_d, K, \beta, K_i$. From figure (8-10), it is inferred that concentration of substrate decreases when k_d, K, K_i increases for some fixed values of other parameter. From figure (11-13), it is inferred that concentration of substrate decreases when α, μ^*, β decreases for some fixed values of other parameter. Figure (14-18) represents the plot of concentration of substrate verses time for different values of $\mu^*, k_d, K, \beta, K_i$. From figure (14-18), it is inferred that concentration of substrate decreases when $k_d, K, K_i, \mu^*, \beta$ increases for some fixed values of other parameter.

4. CONCLUSION:

In this paper, the system of nonlinear differential equations on the concentration of substrate & biomass has been solved analytically. The analytical expressions pertaining to the concentration of substrate & biomass for all values of the parameters are obtained using the New Homotopy Perturbation method. The numerical simulation of Monod and Webb functions shows that the numerical results are in sound agreement with analytical results. This analytical result helps us for the dynamics of the model and to study the correlation between the model parameters.

REFERENCES:

[1] Jump up^ Jana, Amiya K.. Chemical Process Modelling And Computer Simulation, PHI Learning Pvt. Ltd. (2011)

[2] Jump up^ "Bioreactor- Basics"

[3] Rene Alt a,∗, SvetoslavMarkovb, Theoretical and computational studies of some bioreactor models, Computers and Mathematics with Applications 64(2012)350-360.

[4] H. Smith, P. Waltman, The Theory of Chemostat, Cambridge University press, 1995,

[5] J. Monod, The growth of bacterial cultures, Annual Reviews of Microbiology 3 (1949) 371–394.
[6] J.L. Webb, Enzyme and Metabolic Inhibitors, Academic Press, 1963.
[7] He. J.H., Homotopy perturbation technique, Computer methods in Applied Mechanics and Engineering, 178,(1999), 257-262
[8] He J.H (2003) A simple perturbation approach to Blasius Equation", Applied mathematics and computations 140: 217-222.
[9] He J.H. (2006) Some Asymptotic Methods for strongly nonlinear equations. International journal of Modern Physics B 20: 1141-1199.
[10] He J.H (2003) A coupling method of Homotopy technique and a perturbation technique for nonlinear problems. International journal of nonlinear mechanics 35: 37-43.
[11] Ganji D.D, Amini M., Kolahdooz A. (2008) Analytical investigation of hyperbolic Equations via He, methods. American. Journal of Engineering and Applied science.

A STUDY ON DOUBLE LAYERED FUZZY GRAPH

Dr. P. Mahalakshmi

Assistant Professor, Department Of Mathematics
V. H. N. S. N. College (Autonomous),
Virudhunagar-626001, Tamil Nadu, India

ABSTRACT: *In this paper, define a new fuzzy graph named Double Layered Fuzzy Graph (DLFG) and discussed some of its properties using order, size, μ - complement of fuzzy graphs, etc. The concept of connectivity plays an important role in both theory and applications of fuzzy graphs. The relationship between the double layered fuzzy graph and the given fuzzy graph is a cycle are analyzed. Also, this paper generalizes the tree concept in fuzzy labeling graph, which plays an important role in many areas of science and technology.*

Keywords: *Fuzzy graph, Domination in Fuzzy graph, double layered fuzzy graph, domination in double layered fuzzy graphs, Perfect domination.*

1. INTRODUCTION:

The theory of fuzzy sets has been an exponential growth both within mathematics and in its, applications, this ranges from traditional mathematical subjects like logic, topology, algebra, analysis etc. information theory, artificial intelligence, operation research, neural networks and planning etc... Consequently fuzzy set theory has emerged as a potential area of interdisciplinary research and fuzzy graph theory plays a vital role.

Rosenfeld in 1975 considered fuzzy relations on fuzzy sets and developed the theory of fuzzy graph , and then some basic fuzzy graph theoretic concepts and applications have been indicated, many authors found deeper results, and fuzzy analogues of many other graph theoretic concepts, this include fuzzy trees, fuzzy line graphs, operations on fuzzy graphs, automorphism of fuzzy graph, fuzzy interval graphs, cycles and co cycles of fuzzy graphs, bipartite fuzzy graph and metric aspects in fuzzy graph.

2. Fuzzy Sets:

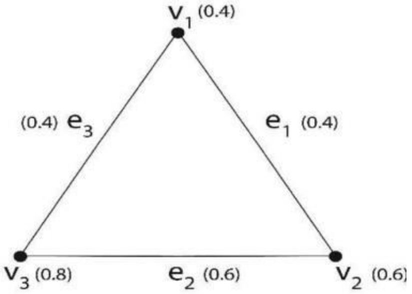
A fuzzy set is a set whose elements have degree of membership. Fuzzy sets are an extension of the classical notion of set (known as a crisp set). A fuzzy set is a pair (A, A) , Where A is a set and A: A → [0,1] for all x ∈ A, (A)(X) is called a grade of membership of x. If A(X) = 1, then x is fully included in (A,A) and n_i if x is not included in (A,A).
If there exists some x ∈A. Such that A(X) =1, then say that (A,A) is normal. Otherwise, we say that (A,A) is subnormal.

In general a fuzzy set is denoted as $A=A(X_1)/X_1+ (X_n)/x_n$ which belongs to a finite universal set. If A(xi)/xi (a singleton) is a pair then it is said to be a “grade of member ship element”.
Complete Bipartite Graphs:

A complete bipartite graph is a bipartite graph G= (V,E) where $v=v_1 v_2$, such that for any two vertices v_1 V_1 and v_2 V_2 ,v_1 ,v_2 is an edge in G. The complete bipartite graph with partitions $|v_1|=m$ and $|v_2|=n$ is denoted by $K_{m, n}$.

Fuzzy Graphs

A fuzzy graph $G = (V, \sigma, \mu)$ is a triple consisting of a nonempty set V together with a pair of functions $\sigma : V \rightarrow [0, 1]$ and $\mu : E \rightarrow [0, 1]$ such that for all $x, y \in V$, $\mu(xy) \leq \sigma(x) \wedge \sigma(y)$. The fuzzy set σ is called the fuzzy vertex set of G and μ the fuzzy edge set of G.



Complete fuzzy Graph:

A fuzzy graph ˜G = (σ, μ) is said to be complete if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$, for all $u, v \in V$ and is denoted by $K\sigma$.

Example :

Let ˜G be fuzzy graph Define ˜G = (σ, μ) by σ(u)=0.8, σ(v)=0.9, σ(w)=0.7, σ(x)=0.6, and μ(u, v)=0.8, μ(v, w)=0.7, μ(w, x)=0.6, μ(x, v)=0.6. Then ˜ G = (σ, μ) is complete fuzzy graph

3. A complete fuzzy graph (K3)

To find double layered complete fuzzy graph

Let $v_1=0.4$, $v_2=0.6$,$v_3=0.8$, $e_1=0.4$, $e_2=0.6$,$e_3=0.4$ be an edge set.

By definition for fuzzy graph

$(x, y) \leq (x) \wedge (y) = \min((x), (y))$

$(v_1,v_2) = \min (0.4, 0.6) = 0.4$

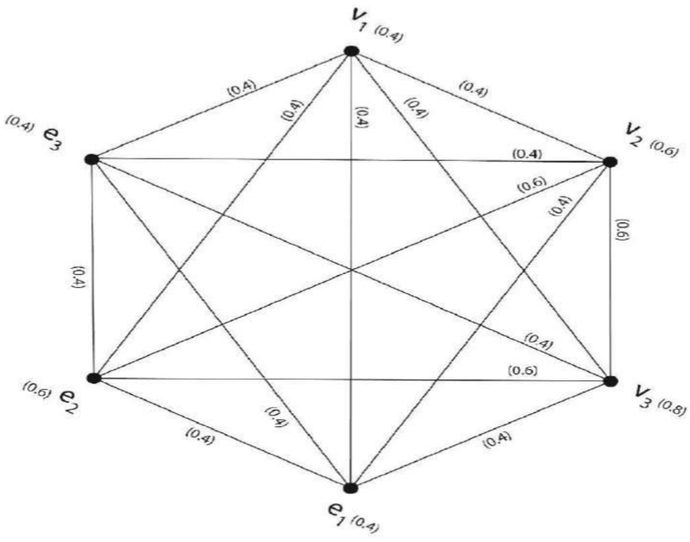
$(v_2, v_3) = \min (0.6, 0.8) = 0.6$

$(v_3, e_1) = \min (0.8, 0.4) = 0.4$

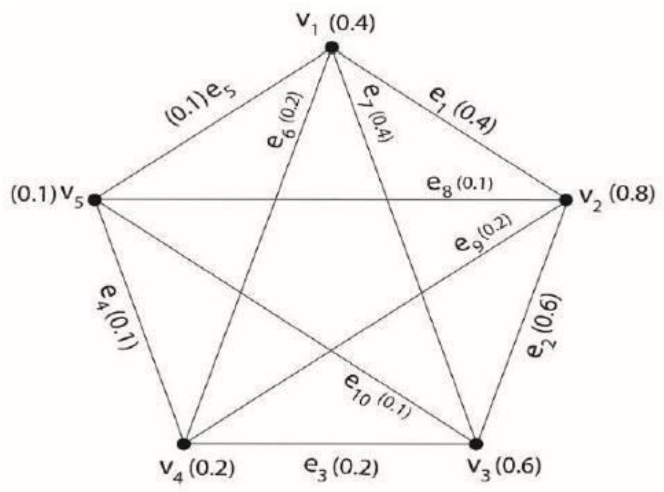
$(e_1, e_2) = \min (0.4, 0.6) =0.4$

$(e_2, e_3) = \min (0.6, 0.4) = 0.4$

$(e_3, e_1) = \min (0.4, 0.4) = 0.4$

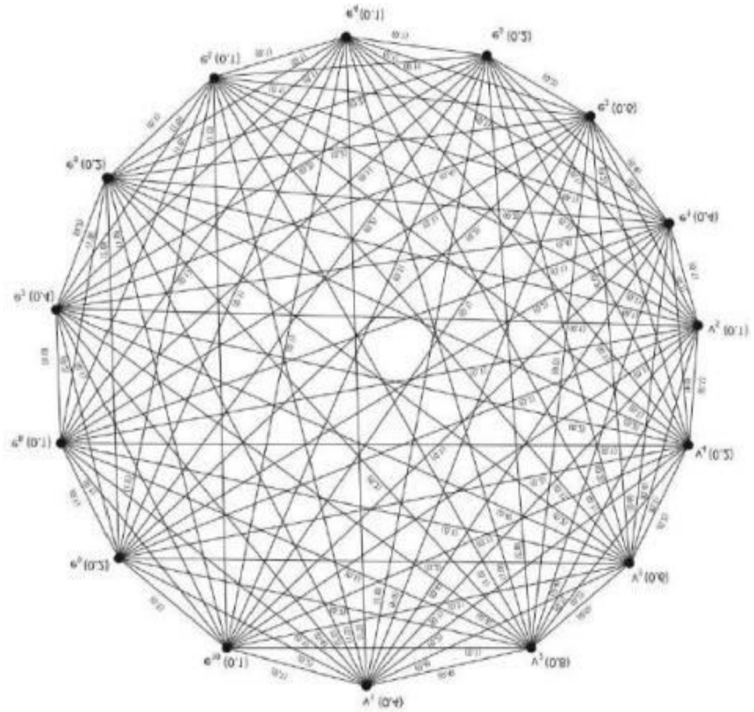


Consider the complete fuzzy graph with vertex 5 ,(K5)



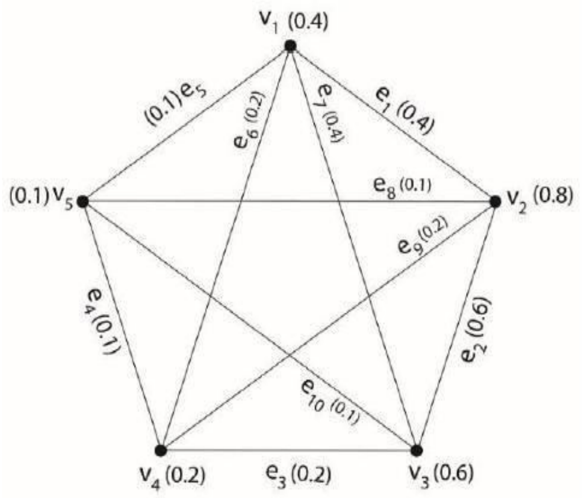
4. A complete fuzzygraph (K5)

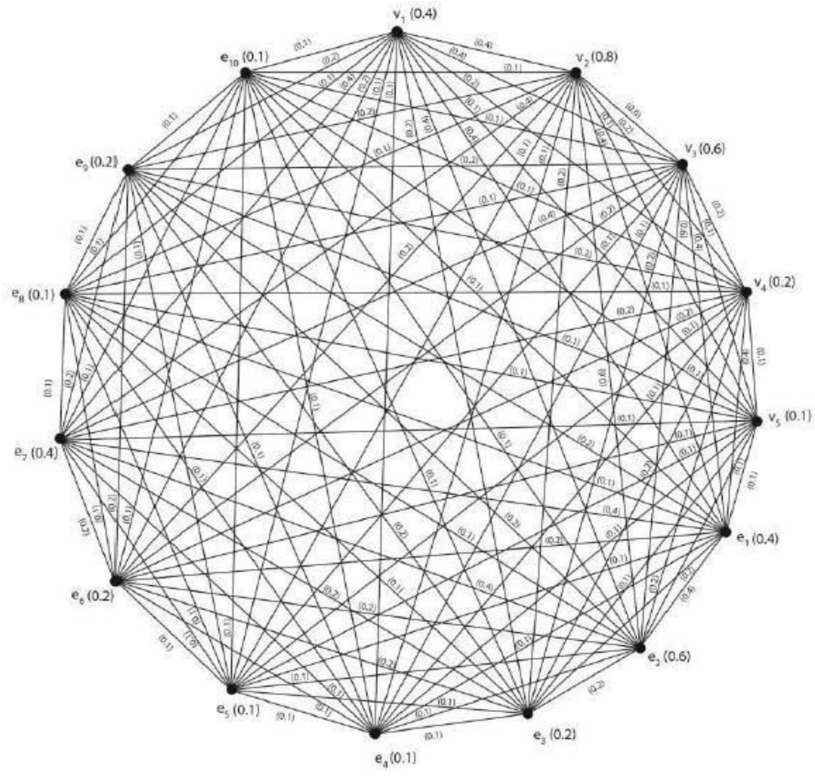
DLCFG of K5



The conversion of complete fuzzy graph into double layered complete fuzzy graph is given as the complete fuzzy graph with vertex5 is(K5)

A complete fuzzy graph(K5)





DLCFG of K_5

Conversion of complete fuzzy graph into double layered complete fuzzy graph

Complete Fuzzy Graph	Double Layered Complete Fuzzy Graph
K3	DLCFG(K_3) = K6
K4	DLCFG(K_4) = K10
K5	DLCFG(K_5) = K15
K6	DLCFG(K_6) = K21
K7	DLCFG(K_7) = K28

K8	DLCFG(K_8) = K36
K9	DLCFG(K_9) = K45
K10	DLCFG(K_{10}) = K55
K11	DLCFG(K_{11}) = K66
K12	DLCFG(K_{12}) = K78
K13	DLCFG(K_{13}) = K91
K14	DLCFG(K_{14}) = K105
K15	DLCFG(K_{15}) = K120
K16	DLCFG(K_{16}) = K136
K17	DLCFG(K_{17}) = K154
K18	DLCFG(K_{18}) = K173
K19	DLCFG(K_{19}) = K192
K20	DLCFG(K_{20}) = K212
K21	DLCFG(K_{21}) =K233
K22	DLCFG(K_{22}) =K255

Theorem 1.1.
If $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ are complete fuzzy graphs, then $G_1 \sqcap G_2$ is complete.
Proof:
If $(u_1, v_1)(u_2, v_2) \in E$, then G_1 and G_2 are complete
and $(\mu_1 \sqcap \mu_2)((u_1, v_1)(u_2, v_2)) = \mu_1(u_1,u_2) \wedge \mu_2(v_1, v_2) = \sigma_1(u_1) \wedge \sigma_1(u_2) \wedge \sigma_2(v_1) \wedge \sigma_2(v_2)$
$= (\sigma_1 \sqcap \sigma_2)((u_1, v_1)) \wedge (\sigma_1 \sqcap \sigma_2)((u_2, v_2))$.
Hence, $G_1 \sqcap G_2$ is complete.

Theorem 1.2
If $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ are complete fuzzy graphs, then $G_1 \bullet G_2$ is complete.
Proof :
If (u,v_1) and $(u,v_2) \in E$,
then $(\mu_1 \bullet \mu_2)((u,v_1)(u,v_2)) = \sigma_1(u) \wedge \mu_2(v_1,v_2) = \sigma_1(u_1)\wedge\sigma_2(v_1)\wedge\sigma_2(v_2)$ (since G_2 is complete)
$= (\sigma_1 \bullet \sigma_2)((u,v_1)) \wedge (\sigma_1 \bullet \sigma_2)((u,v_2))$.
If $(u_1,v_1)(u_2, v_2) \in E$, then G_1 and G_2 are complete
If $(\mu_1 \bullet \mu_2)((u_1,v_1)(u_2,v_2)) = \mu_1(u_1,u_2) \wedge\mu_2(v_1,v_2)$
$= \sigma_1(u_1) \wedge \sigma_1(u_2) \wedge \sigma_2(v_1) \wedge \sigma_2(v_2)$
$= (\sigma_1 \bullet \sigma_2)((u_1, v_1)) \wedge (\sigma_1 \bullet \sigma_2)((u_2, v_2))$.
Hence, $G_1 \bullet G_2$ is complete.

5. CONCLUSION :

Fuzzy graphs have numerous applications in different parts of Science and Engineering like broadcast communications, producing, Social Network, man-made reasoning, data hypothesis, neural systems etc.

REFERENCES:

[1] A.Nagoorgani and J.Malarvizhi, “Some aspects of neighbourhood fuzzy 6646 J. Jon Arockiaraj and V. Chandrasekaran graph”, Inter. Journal of Bulletin Pure and Applied Sciences, 29E (2010) 327 – 333.

[2] A.Nagoorgani and J.Malarvizhi, “Properties of μ - complement of a fuzzy graph”, Inter. Journal of Algorithms, Computing and Mathematics, 2(3) (2009) 73 - 83.

[3] M.S.Sunitha and A.Vijayakumar, “Complement of a fuzzy graph”, Indian Journal of Pure and Applied mathematics, 33(9) (2002) 1451-1464.

[4] A.Rosenfeld, “Fuzzy graphs, in: L.A.Zadeh, K.S. Fu, K. Tanaka and M. Shimura,(editors), Fuzzy sets and its application to cognitive and decision process”, Academic press, New York (1975) 77 – 95.

[5] A.Nagoorgani and K.Radha, “The degree of a vertex in some fuzzy graphs”, Inter. Journal of Algorithms, Computing and Mathematics, 2(3) (2009) 107 - 116.

A STUDY ON UNIFORMITY OF BP-ALGEBRAS

V. Abisha[1] and N.Kandaraj[2]

[1]Research Scholar, PG and Research Department of Mathematics,
Saiva Bhanu Kshatriya College (Affiliated to Madurai Kamaraj University,Madurai),
Aruppukottai-626 101. Tamil Nadu, India.
E-Mail: abisha1997@gmail.com

[2]Associate Professor, PG and Research Department of Mathematics,
Saiva Bhanu Kshatriya College (Affiliated to Madurai Kamaraj University,Madurai),
Aruppukottai-626 101, Tamil Nadu, India.
E-Mail: n.kandarajsbkc1998@gmail.com

ABSTRACT: *In this paper, we define the uniformity on BP-algebras and show how to connect uniform topology with the BP-ideals on BP-algebras. We prove that it is natural for BP-algebras to be topological BP-algebras. Moreover, we find some properties of this structure. Also we explain the uniformity condition of BP-algebras with examples and how it induces the topology on BP-algebras.*

Keywords: *BP-algebras, Uniformity, BP-ideal, Topological BP-algebras.*

Mathematical Subject Classification (2010): 03G25, 06F35, 46J05, 46H05.

1. INTRODUCTION

The two classes of abstract algebras namely BCK-algebras and BCI-algebras were introduced by Imai Y and Iseki K [6]. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. Hu Q P and Li X [5] introduced a wide class of abstract algebras: BCH-algebras. Also it is known that the class of BCI-algebras is a proper subclass of the class of BCH-algebras. Ahn S.S and Han J.S [1] introduced the concepts of BP-algebras and they discussed some relations with BF-algebras. Alo R and Deeba E [3] attempted to study the topological concepts of the BCK-structure. Ahn S.S and Kwon S H [2] studied the topological properties in BCC-algebras. Dudeck W A and Zhang X [4] discussed on ideals and congruence in BCC-algebras. In 2017, Jansi M and Thiruveni V [7] studied the topological structures on BCH-Algebras. In 2019, they [8] also introduced topological BCH-groups. Recently, Complementary Role of Ideals in TSBF-algebras was discussed by Jansi M and Thiruveni V [9]. Nagamani N and Kandaraj N [10, 11] discussed the topological concepts and structures on d-algebras.

Motivated by this, in this paper, we study the issue of attaching topologies to BP-algebras in as natural a manner as possible. We may use the class of BP-ideals of a BP-algebras as the underlying structure whence a certain uniformity and hence a topology is derived, which provides a natural connection between the concept of BP-algebras and the concept of topology. Thus a BP-algebra becomes a topological BP-algebra.

2. PRELIMINARIES :

Definition 2.1 [1]. Let X be a set with a binary operation $*$ and a constant 0. Then $(X,*, 0)$ is called a BP-algebra if it satisfies the following axioms.

1. $x* x = 0$
2. $x* (x * y) = y$
3. $(x* z) * (y * z) = x * y$ for any x, y, z $\in X$.

Proposition 2.2 [1]. If $(X,*, 0)$ is a BP-algebra, then the following results are hold:

For any x, y $\in X$

1. $0 * (0 * x) = x$.
2. $x* (x* y) = y$.
3. $x* 0 = x$.
4. $x* y = 0$ implies $y * x = 0$.
5. $0 * x = 0 * y$ implies $x = y$.
6. $(x* z) * (y * z) = (x * y)$
7. $0 * x = x$ implies $x * y = y * x$

Proposition 2.3 [1]. If $(X, *, 0)$ is a BP-algebra with $(x * y) * z = x * (z * y)$ for any x, y, z $\in X$, then $0 * x = x$ for any $x \in X$.

Theorem 2.4 [1]. If $(X, *, 0)$ is a BP-algebra with $x * y = 0$ and $y * x = 0$, then $x = y$.

Definition 2.5 [4]. Let S be a non-empty subset of a BP-algebra X, then S is called BP-subalgebra of X if $x * y \in S$ for all x, y $\in$ S.

Definition 2.6 [4]. Let $(X,*, 0)$ be a BP-algebra and I be a non-empty subset of X. Then I is called an ideal of X, if it satisfies the following conditions.

1.$0 \in I$.

2. $x * y \in I$ and $y \in I \Longrightarrow x \in I$.

Definition 2.7 [4]. Let $(X, *, 0)$ be a BP-algebra and I be a non-empty subset of X. Then I is called a BP-ideal of X if it satisfies the following conditions:

1.$0 \in I$.

2. $(x * y) * z \in I$ and $y \in I \Longrightarrow x * z \in I$.

Lemma 2.8 [4]. In a BP-algebra X any BP-ideal I is an ideal in X.

Remark2.9 [4]. Any BP-ideal of a BP-algebra is subalgebra, but converse is not true.

2. Any ideal of a BP-algebra is subalgebra, but converse is not true.

Definition 2.10 [2]. Let X be a BP-algebra. An equivalence relation $\sim$ on X is called a left congruence if $x\sim y$ implies $u * x \sim u * y$, where x, y, u $\in$ X.

An equivalence relation $\sim$ on X is called a right congruence if $x\sim y$ implies $x * u \sim y * u$, where x, y, u $\in$ X.

Definition 2.11[2]. Let X be a BP-algebra. An equivalence relation $\sim$ on X is called a congruence if $x\sim y$, $u \sim v$ imply $x * u \sim y * v$, where x, y, u, v $\in$ X.

Proposition 2.12[2]. Let X be a BP-algebra and $\sim$ be an equivalence relation on X. Then $\sim$ is congruence if and only if it is both a left congruence and a right congruence.

Definition 2.13[2]. Let $(X, *, 0)$ be a BP-algebra. We can define a binary relation "$\leq$" by $x \leq y$ if and only if $x * y = 0$, is called a BP-order on X. Then it is easy to show that $\leq$ is a partial order on X

Theorem 2.14 [7]. Let X be a set and $S \subseteq P(X \times X)$ be a family such that for every $U \in S$ the following conditions hold:

(a). $\triangle \subseteq U$

(b). U^{-1} contains a member of S.

(c). there exists a $V \in S$ such that $V \circ V \subseteq U$. Then there exists a unique uniformity u, for which S is a subbase.

3. UNIFORMITY ON BP-ALGEBRAS :

In this section we introduce the uniformity condition on BP-algebras with example and how it induces the topology on BP-algebras.

Definition3.1: Let B be a BP-algebra and U and V be any subsets of $B \times B$. Define

$X \circ Y = \{(a, b) \in B \times B \,/\, \text{for some } c \in B, (a, c) \in X \text{ and } (c, b) \in Y\}$,

$X^{-1} = \{(a, b) \in B \times B \,/\, (b, a) \in X\}$,

$\nabla = \{(a, a) \,/\, a \in B\}$.

Example 3.2: Consider a BP-algebra (B = {0, p, q, r},∗, 0) with Cayley table

∗	0	p	q	r
0	0	q	p	r
p	p	0	r	q
q	q	r	0	p
r	r	p	q	0

Let X = {(0, 0), (q, 0)} and Y = {(0, r), (0, q)}

$X \circ Y$ = {(0, q), (0, r), (q, q), (q, r)},

X^{-1} = {(0, 0), (0, q)},

∇ = {(0, 0), (p, p), (q, q)}.

Definition 3.3: Let (B,∗, 0) be a BP-algebra. A non-empty collection $\mathbb{K}$ of subsets of $B \times B$ is called uniformity on B if it satisfies the following axioms.

(U1) $\nabla \subseteq X$ for any $X \in \mathbb{K}$,

(U2) If $X \in \mathbb{K}$, then $X^{-1} \in \mathbb{K}$,

(U3) If $X \in \mathbb{K}$, then there exist a $Y \in \mathbb{K}$ such that $Y \circ Y \subseteq X$,

(U4) If $X, Y \in \mathbb{K}$, then $X \cap Y \in \mathbb{K}$,

(U5) If $X \in \mathbb{K}$ and $X \subseteq Y \subseteq B \times B$, then $Y \in \mathbb{K}$.

The pair $(B, \mathbb{K})$ is called a uniform structure.

Example 3.4: Let (B = {p, 0}, ∗, 0) be a BP-algebra.

Define $\mathbb{K}$ = {{(0, p), (p, p)}, {(0, 0), (p, p), (p, 0)}, {(0, 0), (p, p), (0, p)},
{(0, 0), (p, p), (p, 0), (0, p)}}.

The pair $(B,\mathbb{K})$ is a uniform structure.

Theorem 3.5: If I is an ideal of a BP-algebra B, then the relation defined on B by $a \sim_I b$ if and only if $a * b,\ b * a \in I$ is an equivalence relation on B.

Proof: Let I be an ideal of a BP-algebra B.

Reflexive:

Clearly the relation $\sim_I$ is reflexive.

Symmetric:
Since $a * a = 0$ and $0 \in I$
If $a \sim_I b$ implies $a * b,\ b * a \in I$
$\Longrightarrow b * a,\ a * b \in I \Longrightarrow b \sim_I a$
The relation $\sim_I$ is symmetric.
Transitive:
If $a \sim_I b$ and $b \sim_I c \Longrightarrow a * b,\ b * a \in I$ and $b * c,\ c * b \in I$.
Since I is an ideal, $(a * b) * (c * b) = (a * c) \in I$. (By proposition 2.2(6))
Similarly $(c * b) * (a * b) = (c * a) \in I$.
Therefore the relation $\sim_I$ is transitive. Hence the relation $\sim_I$ is an equivalence relation.

Definition 3.6: Let $(B, *, 0)$ be a BP-algebra, then the Congruence relation on B is an equivalence relation $\cong$ on the elements of B satisfying $h_1 \cong h_2$ and $f_1 \cong f_2$

$$\Longrightarrow h_1 * f_1 = h_2 * f_2 \text{ for all } h_1, h_2, f_1, f_2 \in B.$$

Theorem 3.7: Let B be a BP-algebra and I is an ideal on B, then the relation $\sim_I$ is a Congruence relation on B.

Proof. From theorem 3.5, the relation $\sim_I$ is an equivalence relation.
It is enough to prove that , if $a \sim_I b$ and $f \sim_I n$, then $a * f \sim_I b * n$.
Since $a \sim_I b$ and $f \sim_I n$, then $a * b, b * a, f * n, n * f \in I$.
To prove $(a * f) * (b * n)$ and $(b * n) * (a * f) \in I$.
Consider $((a * f) * (b * n)) * (n * f) = ((a * f) * (n * f)) * (b * n)$
$= (a * n) * (b * n)$
$= (a * b) \in I$ (By proposition)
Since I is an ideal in B, $(a * f) * (b * n) \in I$
Similarly, we can prove $(b * n) * (a * f) \in I$
Hence the relation $\sim_I$ is a Congruence relation on B.

Theorem 3.8. Let I be an ideal of a BP-algebra B.
We define $X_I = \{(a, b) \in B \times B \,/\, a * b \in I \text{ and } b * a \in I\}$ and let $\mathbb{K}^+ = \{U_I \,/\, I \text{ is an ideal of } B\}$.
Then $\mathbb{K}^+$ satisfies the axioms U1 to U4.

Proof. [U1]. Let $(a, a) \in \nabla$, since $a * a = 0 \in I$, $(a * a) \in X_I$
Hence $\nabla \subseteq X_I$ for any $X_I \in \mathbb{K}^+$
[U2]. For any $X_I \in \mathbb{K}^+$
$(a, b) \in X_I \Leftrightarrow a * b \in I$ and $b * a \in I$
$\Leftrightarrow b \sim_I a$
$\Leftrightarrow (b. a) \in X_I$
$\Leftrightarrow (a. b) \in X_I^{-1}$
Hence $X_I^{-1} = X_I \in \mathbb{K}^+$
[U3]. For any $X_I \in \mathbb{K}^+$, the transitivity condition of $\sim_I$ implies that $X_I \circ X_I \subseteq X_I$
[U4]. For any X_M and $X_N \in \mathbb{K}^+$
To prove $X_M \cap X_N \in \mathbb{K}^+$
$(a, b) \in X_M \cap X_N \Leftrightarrow (a, b) \in X_M$ and $(a, b) \in X_N$
$\Leftrightarrow a * b,\ b * a \in M \cap N$
$\Leftrightarrow a \sim_{M \cap N} b$
$\Leftrightarrow (a,b) \in X_{M \cap N}$

Since M ∩ N is an ideal of BP-algebra, $X_M \cap X_N = X_{M \cap N} \in \mathbb{K}^+$

Hence the theorem.

Theorem 3.9: Let $\mathbb{K} = \{X \subseteq B \times B \,/\, X_I \subseteq X$ for some $X_I \in \mathbb{K}^+\}$. Then $\mathbb{K}$ satisfies the axioms for a uniformity on BP-algebra B and hence the pair (B, $\mathbb{K}$) is a uniform structure.

Proof. By theorem 3.8, the collection $\mathbb{K}$ satisfies the axioms U1 to U4.

It is enough to prove that $\mathbb{K}$ satisfies U5.

Let $X \in \mathbb{K}$ and $X \subseteq Y \subseteq B \times B$, then there exist a $X_I \subseteq X \subseteq Y$.

This means that $Y \in \mathbb{K}$.

Hence the theorem.

Notation 3.10: Let B be a BP-algebra, $a \in B$ and $X \in \mathbb{K}$.

Define $X[a] = \{b \in B \,/\, (a, b) \in X\}$.

Theorem 3.11: Let B be a BP-algebra. Then

$T = \{G \subseteq B / \forall\, a \in G$, there exist $X \in \mathbb{K}$, $X[a] \subseteq G\}$ is a topology on B

Proof: Since $\emptyset$ and the set B belongs to T.

From the definition, clearly T is closed under arbitrary unions.

Finally we prove that T is closed under the finite intersection.

Let G, H belongs to T and suppose $a \in G \cap H$, then there exists X and $Y \in \mathbb{K}$ such that $X[a] \subseteq G$ and $Y[a] \subseteq H$

Let $U = X \cap Y$, then $U \in \mathbb{K}$

Also $U[a] \subseteq X[a] \cap Y[a]$ and so $U[a] \subseteq G \cap H$

Therefore $G \cap H \in T$.

Thus T is a topology of B.

Hence the theorem.

Definition 3.12: Let B be a BP-algebra. For any $a \in B$, X [B] is an open neighborhood of a.

Example 3.13: Let B = {0, p, q} be a non-empty set and the collection

$\mathbb{K} = \{\{\nabla, (p, 0), (0, p), (0, q), (q, 0), (q, p)\}, \{\nabla, (p, 0), (0, p), (0, q), (q, 0), (p, q)\}\}$ is a uniform structure.

Define a topology $T = \{B, \emptyset, \{p, 0\}, \{q\}\}$ (By using 3.9)

Then T is called the uniform topology on B induced by $\mathbb{K}$

Example 3.14: Let B = {0, p, q, r} be a BP-algebra with the Cayley table given below.

∗	0	p	q
0	0	q	p
p	p	0	q
q	q	p	0

It is easy to prove that A = {0, P}, E = {0, q}, D = {0} and B are the only ideals in B.

We can see that $X_A = \nabla \cup \{(0, p), (p, 0)\}$

$X_E = \nabla \cup \{(q, 0), (0, q)\}$, $X_D = \nabla$ and $X_B = B \times B$

Therefore $\mathbb{K}^+ = \{X_D, X_E, X_A, X_B\}$ and

$\mathbb{K} = \{X \subseteq B \times B \,/\, X_A \subseteq X$ for some $X_A \in \mathbb{K}^+\}$.

If $X = X_A$, then X [0] = X[p] = {0, p}

Therefore $T = \{C \subseteq B, \forall\, a \in C$, there exist $X \in \mathbb{K}$, $X[a] \subseteq C\} \supseteq \{B, \emptyset, \{q\}, \{0, p\}\}$.

Since $\{B, \emptyset, \{q\}, \{0, p\}\}$ is a topology on B, the topology Ton B induced by an ideal.

$A = \{0, p\}$ relative to X_A
Let $A = \{0\}$, then $X[a] = \{a\}\ \forall\ a \in B$ and we define $T = 2^a$, the discrete topology
Moreover, if we consider B as an ideal of B, then $X[a] = B$, for all $a \in B$ and
we get $T = \{\emptyset, B\}$, the indiscrete topology. $a*b$

Theorem 3.15: Let I be a BP-ideal of a BP-algebra B. If we define a binary operation on the quotient set $B/I = \{I_a\ /\ a \in B\}$ by $I_a * I_b = I_{a*b}$, then $(B/I, *, I_0)$ is a BP-algebra called the Quotient algebra of B relative to I.

Proof. If $I_a = I_{a1}$ and $I_b = I_{b1}$, then $a \sim a^1$ and $b \sim b^1$
Hence $\sim$ is a congruence relation.
Therefore $I_a * I_b = I_{a*b} = = I_{a1*b1} = I_{a1} * I_{b1}$
Thus $*$ is well defined on B/I.
Assume that $I_a * I_b = I_b * I_a = I_0$, then $Ia_{*b} = I_{b*a} = I_0$
Hence $a * b \sim 0$ and $b * a \sim 0$.
Therefore $(B/I, *, I_0)$ is a edge BP-algebra
By the proposition 2.2 (2) and (6), we have $(B/I, *, I_0)$ is a BP-algebra.

4. CONCLUSION:

S.S. Ahn and J.S. Han [1] introduced the concept of BP-algebras, which is generalization of B-algebras. In this paper we show that how to connect the topology concepts with BP-algebras.

Acknowledgement
The authors would like highly grateful to referees for their valuable suggestions and comments which improved the paper.

Competing Interests
The authors declare that they have no competing interests.

Authors Contribution
All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

REFERENCES:

[1] S.S. Ahn and J.S. Han: On BP-algebras, Hacettepe Journal of Mathematics and Statistics, Volume 42(5), (2013), 551-557.
[2] S.S. Ahn and S.H. Kwon: Topological properties in BCC-algebras, Communications of the Korean Mathematical Society 23 (2008), No.2, pp.169-178.
[3] R.A. Alo and E.Y. Deeba: Topologies of BCK-algebras, Mathematica Japonica 31 (No.6) (1986), 841-853. Corpus ID: 228270402
[4] W.A. Dudeck and X. Zhang: On ideals and Congruences in BCC-algebras, Czechoslovak Mathematical Journal, 48(123) (1998), 21-29.
[5] Q.P. Hu and X. Li: On BCH-algebras, Mathematics Seminar Notes (Kobe University) Vol. 11, No. 2, 1983, pp. 313-320.
[6] Y. Imai and K. Iseki: On axiom system of propositional calculi XIV, Proc. Japan Academy 42 (1966), 19-22.
[7] M. Jansi and V. Thiruveni, Topological Structures on BCH-Algebras, International Journal of Innovative Research in Science, Engineering and Technology, Vol.6, Issue 12,

December 2017.
[8] M. Jansi and V. Thiruveni, Topological BCH-groups, Malaya Journal of Matematik, Vol.1, 83-85, 2019.
[9] M. Jansi and V. Thiruveni, Complementary Role of Ideals in TSBF-algebras, Malaya Journal of Matematik, Vol.8, No. 3, 1037-1040, 2020,
[10] N. Nagamani and N. Kandaraj, Topological structures on d-algebras, Journal of Physical Sciences, Vol.24, June 2019, 9-19.
[11] N. Nagamani and N. Kandaraj, Topological concepts on d-algebras, International Journal of Research and Analytical Reviews, Volume 6, Issue 1, February 2019, 643-649.
[12] B.T. Sims, Fundamentals of Topology, Macmillan Publishing Co., Inc., New York, 1976.

(α, β) - REVERSE DERIVATIONS ON PRIME AND SEMIPRIME SEMIRINGS

Dr. U. Revathy
Assistant Professor, Department of Mathematics,
Thiruvalluvar College, Papanasam,
Tamil Nadu, India.
E-mail: revaulagu@gmail.com

ABSTRACT: *Motivated by some results on (α, β) – reverse derivations on prime and semiprime rings in [4]. The authors investigated some properties of (α, β) – reverse derivations on prime and semiprime rings. The main results of that paper is if R is a prime ring of characteristic $\neq 2$, (α, β) - reverse derivation and generalized (α, β) – reverse derivation are (α, β) derivations and generalized (α, β)derivations of R, respectively and also derived some necessary and sufficient condition for (α, β) – reverse derivations exist. Now in this paper I also investigate same thing in prime and semiprime semiring.*

Keywords: *Semiring, Prime, Semiprime, Reverse Derivation, Generalized Reverse Derivation, Generalized (α, β) – reverse derivation.*

1. INTRODUCTION:

A Semiring (S,+,•) is a non-empty set S together with two binary operations, + and • such that, i) (S,+) is monoid and (S,•) is semigroup ii) For all a, b, c ∈ S, a . (b + c) = a . b + a . c and (b + c) . a = b . a + c . a. A semiring S is said to be n - torsion free if nx = 0 ⇒ x = 0, ∀x ∈ S. A semiring S is Prime if xSy = 0 ⇒ x = 0 or y = 0, ∀ x, y ∈ S and S is Semi Prime if x S x = 0 ⇒ x = 0, ∀ x ∈ S.

For $x, y \in S$, $xy - yx$ is denoted by [x, y] and $x\alpha(y) - \beta(y)x$ is denoted by $[x, y]_{\alpha,\beta}$

An additive mapping $d: S \to S$ is called a derivation if d(x y) = d(x) y + x d(y) , ∀ x,y ∈ S. For a fixed $a \in S$, $I_a: S \to S$ is given by $I_a(x) = [a, x]$, is called an inner derivation determined by a.

An additive mapping $D : S \to S$ is called a generalized derivation if there exist a derivation d of S such that $D(xy) = D(x)y + x\,d(y), for\ all\ x, y \in S$. $C_{\alpha,\beta} = \{c \in S\ /\ c\alpha(s) = \beta(s)c\ ,\ for\ all\ s \in S\}$ is known as (α, β) - center of S. An additive mapping $d: S \to S$ is called an (α, β) – derivation if $d(xy) = d(x)\alpha(y) + \beta(x)d(y)$, for all x,y ∈ S. An additive mapping $D: S \to S$ is said to be a generalized (α, β) – derivation associated with (α, β) – derivation d if $D(xy) = D(x)\alpha(y) + \beta(x)d(y), for\ all\ x, y \in S$. For fixed $a \in S$, $I_a: S \to S$ is given by $I_a(x) = [a, x]_{\alpha,\beta}$ which is called (α, β) – inner derivation determined by a.

Throughout this paper, S is a semiring, Z(S) is the center of S, α, β are homomorphisms of S and $C_\alpha = \{c \in S / c\alpha(x) = \beta(x)c, forall\ x \in S\}$. We use the basic commutator identities.
i) [x,yz] = y [x,z] + [x,y]z ii) $[x,yz]_{\alpha,\beta} = [x,y]_{\alpha,\beta}\, \alpha(z) + \beta(y)\, [x,z]_{\alpha,\beta}$

The main result of this paper is for a semiprime semiring S, any (α,β) – reverse derivation is a (α,β) – derivation mapping S into the center. The another main result of this paper is , if S is a prime semiring, D is a non-zero (α,β) – reverse derivation of S, then D is a (α,β) – derivation of S and if D is a nonzero generalized (α,β)- reverse derivation of S, then D is a generalized (α,β) – derivation of S.

2. (α,β) – REVERSE DERIVATION ON PRIME SEMIRING

Definition : 2.1

An additive mapping $D : S \to S$ is said to be an (α,β) – reverse derivation of S if $D(xy) = D(y)\alpha(x) + \beta(y)D(x), \forall x,y \in S.$

Definition : 2.2

Let d be a (α,β) – reverse derivation. An additive mapping $D: S \to S$ is said to be a generalized (α,β) – reverse derivation associated with d if $D(xy) = D(y)\alpha(x) + \beta(y)\, d(x)$, $\forall x,y \in S$

Theorem : 2.3

Let S be a prime semiring and β be a automorphisms of S. A mapping D on S is a non-zero (α,β) – reverse derivation of S iff S is commutative and D is an ordinary (α,β) – derivation of S.

Proof:

Let S be a prime semiring and α,β be a automorphisms of S.

Assume that D is a (α,β) – reverse derivation of S.

Let $x,y,z \in S$. Then $D(x(yz)) = D(yz)\alpha(x) + \beta(yz)D(x)$

$= D(z)\alpha(y)\alpha(x) + \beta(z)D(y)\alpha(x) + \beta(y)\beta(z)D(x)$ -----(1)

Also $D\big((xy)z\big) = D(z)\alpha(x)\alpha(y) + \beta(z)D(y)\alpha(x) + \beta(z)\beta(y)D(x)$ --------------------(2)

From (1) and (2), we get $D(z)\alpha([x,y]) + \beta([z,y])D(x) = 0$ -----------------------------(3)

Put y = x, $\beta([z,x])D(x) = 0, \forall\, x,z \in S$ -----------------------------(4)

Replace z by zy in (4), $\beta(z)\beta([y,x])D(x) + \beta([z,x])\beta(y)D(x) = 0$

Using (4), we get $\beta([z,x])\beta(y)D(x) = 0\, , \forall\, x,y,z \in S$

Since S is prime, $x \in Z(S) or\ D(x) = 0, \forall\, x \in S$

Let $A = \{x \in S / x \in Z(S)\}$ and $B = \{x \in S / D(x) = 0\}$. Clearly A and B are additive subgroups of S such that $S = A \cup B$. We know that the union of subgroups is subgroup iff one is contained in the other. Therefore S = A or S = B.

If S = B, then D = 0.

$\Rightarrow\Leftarrow$ to our assumption

$\therefore S = A$

So S is commutative, $D(xy) = D(yx) = D(x)\alpha(y) + \beta(x)D(y)$

Hence D is an (α,β)- derivation.

Example: 1 Consider the Semiring $M_2(S) = \left\{\begin{pmatrix} a & 0 \\ b & a \end{pmatrix} / a,b \in S\right\}$. Define $D: S \to S$ by $D(x) = \begin{pmatrix} 0 & 0 \\ b & 0 \end{pmatrix}$, $\alpha(x) = \begin{pmatrix} a & 0 \\ b & a \end{pmatrix}$, $\beta(x) = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}$, $\forall x \in M_2(S)$.

It is easy to verify that D is $(\alpha,\beta) -$ reverse derivation and ordinary $(\alpha,\beta) -$ derivaton.

Example : 2

Consider the Semiring $M_2(S) = \left\{\begin{pmatrix} a & 0 \\ b & c \end{pmatrix} / a, b \in S\right\}$. Define $D: S \to S$ by $D(x) = \begin{pmatrix} 0 & 0 \\ b & 0 \end{pmatrix}$, $\alpha(x) = \begin{pmatrix} c & 0 \\ b & a \end{pmatrix}, \beta(x) = \begin{pmatrix} a & 0 \\ 0 & c \end{pmatrix}, \forall x \in M_2(S)$.

It is easy to verify that D is neither $(\alpha, \beta) -$ reverse derivation nor ordinary $(\alpha, \beta) -$ derivaton.

Theorem : 2.4

Let S be a prime semiring and β be automorphisms of S. A mapping D is a non-zero generalized (α, β)-reverse derivation with (α, β)-reverse derivation d of S iff S is commutative and D is an ordinary generalized (α, β) – derivation with a (α, β) – derivation d of S.

Proof:

Let D be a non-zero generalized (α, β)-reverse derivation with (α, β)-reverse derivation d of S. Since d is a (α, β)-reverse derivation, by previous theorem, S is commutative and d is a (α, β)-derivation.

Since S is commutative , $D(xy) = D(yx) = D(x)\alpha(y) + \beta(x)d(y), \forall x, y \in S$

$\therefore$ D is an ordinary generalized (α, β) – derivation of S with (α, β) – derivation d of S.

3. (α, β) – REVERSE DERIVATION ON SEMIPRIME SEMIRING

Lemma: 3.1

Let S be a 2-torsionfree semiprime semiring, $s \in S, \alpha, \beta$ be epimorphisms of S and $D: S \to S$ such that $D(x) = s\alpha(x) + \beta(x)s$. If D is a (α, β) – reverse derivation of S then D = 0 and s = 0.

Proof:

Let S be a 2-torsionfree semiprime semiring, $s \in S, \alpha, \beta$ be epimorphisms of S and $D: S \to S$ such that $D(x) = s\alpha(x) + \beta(x)s$.

For any $x, y \in S$, $D(xy) = s\alpha(xy) + \beta(xy)s$

On the other hand, $D(xy) = D(y)\alpha(x) + \beta(y)D(x)$

$= s\alpha(y)\alpha(x) + \beta(y)s\alpha(x) + \beta(y)s\alpha(x) + \beta(y)\beta(x)s$

$= s\alpha(yx) + 2\beta(y)s\alpha(x) + \beta(yx)s$

$= D(yx) + 2\beta(y)s\alpha(x)$

$\therefore D([x, y]) = 2\beta(y)s\alpha(x), \forall x, y \in S$ -------------- (5)

Similarly, $D([y, x]) = 2\beta(x)s\alpha(y), \forall x, y \in S$

Since $D([x, y]) + D([y, x]) = 0$, $2[\beta(y)s\alpha(x) + \beta(x)s\alpha(y)] = 0$

Since S is 2- torsion free , $\beta(x)s\alpha(y) + \beta(y)s\alpha(x) = 0$ -------------- (6)

Replacing y by yz , $\beta(x)s\alpha(yz) + \beta(yz)s\alpha(x) = 0$

$\beta(x)s\alpha(y)\alpha(z) + \beta(y)\beta(z)s\alpha(x) = 0$

$\beta(x)s\alpha(x)\alpha(z) + \beta(x)\beta(z)s\alpha(x) = 0$

$\beta(x)[s\alpha(x)\alpha(z) + \beta(z)s\alpha(x)] = 0$

Since S is semiprime, $s\alpha(x)\alpha(z) + \beta(z)s\alpha(x) = 0$

$[s\alpha(z) + \beta(z)s]\alpha(x) = 0$

$s\alpha(z) + \beta(z)s = 0, \forall z \in S$

ie, D = 0. (5) gives, $2\beta(y)s\alpha(x) = 0, \forall x, y \in S$

Since S is 2-torsionfree semiprime semiring, c = 0.

Lemma : 3.2

Let S be a semiring, $a, b \in S$, α, β be mappings of S and $D(x) = a\alpha(x) + \beta(x)b$: If D is a (α, β) −reverse derivation of S then the equality $a\big(\alpha(xy) - \alpha(y)\alpha(x)\big) + \beta(xy) - \beta(y)\beta(x))b = \beta(y)(b + a)\alpha(x)$ is satisfied.

Proof:

For any $x, y \in S, D(xy) = a\alpha(xy) + \beta(xy)b$

Since D is (α, β) −reverse derivation of S,

$$D(xy) = D(y)\alpha(x) + \beta(y)D(x)$$

$$= a\alpha(y)\alpha(x) + \beta(y)b\alpha(x) + \beta(y)a\alpha(x) + \beta(y)\beta(x)b$$

$$a\alpha(xy) + \beta(xy)b = a\alpha(y)\alpha(x) + \beta(y)b\alpha(x) + \beta(y)a\alpha(x) + \beta(y)\beta(x)b$$

$$\therefore a\big(\alpha(xy) - \alpha(y)\alpha(x)\big) + \beta(xy) - \beta(y)\beta(x))b = \beta(y)(b + a)\alpha(x)$$

Theorem : 3.3

Let S be a 2-torsinfree semiprime semiring, $a, b \in S, \alpha$ be epimorphism of S, β be automorphism of S and $D: S \to, \ni D(x) = a\alpha(x) + \beta(x)b$. If D is a non-zero $(\alpha, \beta) -$ reverse derivation of S then D is ordinary inner $(\alpha, \beta) -$ derivation of S which is determined by a.

Proof

For any $x, y \in S, D(xy) = a\alpha(xy) + \beta(xy)b$

$$D(yx) = a\alpha(yx) + \beta(yx)b$$

Using Lemma 3.2,

$$\therefore D[x, y] = a\big(\alpha(xy) - \alpha(y)\alpha(x)\big) + \beta(xy) - \beta(y)\beta(x))b = \beta(y)(b + a)\alpha(x)$$

Similarly, $D[y, x] = \beta(x)(b + a)\alpha(y), \forall\, x, y \in S$

D([x,y]) + D ([y,x]) = 0

$\beta(x)(b + a)\alpha(y) + \beta(y)(b + a)\alpha(x) = 0$ ----------------(7)

Replacing y by yz , $\beta(x)(b + a)\alpha(y)\alpha(z) + \beta(y)\beta(z)(b + a)\alpha(x) = 0$

$\beta(x)(b + a)\alpha(x)\alpha(z) + \beta(x)\beta(z)(b + a)\alpha(x) = 0$

$\beta(x)[(b + a)\alpha(z) + \beta(z)(b + a)]\alpha(x) = 0$

Since α, β are epimorphism and S is Semiprime, we get,

$(b + a)\alpha(z) + \beta(z)(b + a) = 0, \forall\, z \in S$

$\beta(y)(b + a) + (b + a)\alpha(y) = 0$ ----------------(8)

(7) implies, $-\beta(x)\beta(y)(b + a) - \beta(y)\beta(x)(b + a) = 0$

$\beta(xy + yx)(b + a) = 0, \forall x, y \in S$ ---------------(9)

Replace y by yz and use the equation x (yz) + (yz) x = y (xz + zx) + [x,y] z

$$\beta(y(xz + zx) + [x, y]z)(b + a) = 0, \forall x, y, z \in S$$

Putting $z = \beta^{-1}(b + a)z[x, y]$,

$$\beta([x, y])(b + a)\beta(z)\beta([x, y]) = 0$$

Since S is semiprime,$\beta([x, y])(b + a) = 0$ ---------------(10)

From (9) and (10), $2\beta(xy)(b + a) = 0, \forall x, y \in S$

Since S is 2-torsion free semiprime semiring, (b+a) = 0

$$D(x) = a\alpha(x) - \beta(x)a = [a, x]_{\alpha,\beta}$$

Hence D is ordinary inner $(\alpha, \beta) -$ derivation of S determined by a.

Theorem : 3.4

Let S be a 2-torsionfree semiprime semiring, $a, b \in S, \alpha$ be anti-epimorphisms of S, β be anti automorphisms of S and $D(x) = a\alpha(x) + \beta(x)b$. If D is a non-zero (α, β)-reverse derivation of S then D is ordinary inner (α, β)- derivation of S which is determined by a.

Proof:

By lemma 3.2, and (α, β) are anti-homomorphisms, $\beta(y)(b + a)\alpha(x) = 0, \forall x, y \in S$

Since S is Semiprime semiring, b + a = 0, using hypothesis, $D(x) = [a, x]_{\alpha,\beta}$

Hence D is ordinary inner $(\alpha, \beta) -$ derivation of S determined by a.

Theorem : 3.5

Let S be a semiprime semiring, α, β be a automorphisms and D and G be $(\alpha, \beta) -$ reverse derivations of S such that, $D(x)\alpha(y) + \beta(y)G(x) = 0, \forall x, y \in S$. Then $D(y)\alpha([z, x]) = \beta([z, x])G(y) = 0, \forall x, y, z \in S$, in particular, D and G map Z(S).

Proof

Let $D(x)\alpha(y) + \beta(y)G(x) = 0, \forall x, y \in S$ ----------------(11)

Put x = x y, $D(xy)\alpha(y) + \beta(y)G(xy) = 0, \forall x, y \in S$

Using $(\alpha, \beta) -$ reverse derivations and (11), we get

$D(x)\alpha(xy) + \beta(y)G(y)\alpha(x) = 0, \forall x, y \in S$

$D(x)\alpha(xy) + \beta(y)G(y)\alpha(x) = D(y)\alpha(xy) - D(y)\alpha(y)\alpha(x) = 0$

$D(y)\alpha([x, y]) = 0, \forall x, y \in S$ ----------------(12)

Put $x = \alpha^{-1}(z)x$,

$D(y)z\alpha([x, y]) = 0, \forall x, y, z \in S$ ----------------(13)

Linearizing (12), $0 = D(y + z)\alpha([x, y + z])$

$= D(y)\alpha([x, y]) + D(y)\alpha([x, z]) + D(z)\alpha([x, y]) + D(z)\alpha([x, z])$

$= D(y)\alpha([x, z]) + D(z)\alpha([x, y])$

Hence, $D(z)\alpha([x, y]) = D(y)\alpha([z, x]), \forall\, x, y, z \in S$

Now to prove $D(y)\alpha([z, x]) = 0$

Consider $D(y)\alpha([z, x])sD(y)\alpha([z, x]), \forall s \in S$

$D(y)\alpha([z, x])sD(y)\alpha([z, x]) = D(y)\alpha([z, x])sD(z)\alpha([x, y])$

Put $t = \alpha([z, x])sD(z)$, we get,

$D(y)\alpha([z, x])sD(y)\alpha([z, x]) = D(y)t\alpha([x, y])$

= 0 [since by (13)

Hence $D(y)\alpha([x, y]) = 0, \forall\, x, y \in S$ ----------------(A)

Next to show that $D(S) \subseteq Z(S)$

Replacing z by $z\alpha^{-1}(D(y)$ in (A), $D(y)\alpha(z)[D(y), \alpha(x)] = 0, \forall x, y, z \in S$ ----------------(14)

Put z = xz, $D(y)\alpha(x)\alpha(z)[D(y), \alpha(x)] = 0, \forall x, y, z \in S$

Multiply $\alpha(x)$ in (14), $\alpha(x)D(y)\alpha(z)[D(y), \alpha(x)] = 0$ ----------------(15)

Subtract (14) and (15), $[D(y), \alpha(x)]\alpha(z)[D(y), \alpha(x)] = 0$

Since S is Semiprime, $[D(y), \alpha(x)] = 0, \forall x, y \in S$

Hence $D(S) \subseteq Z(S)$

Similarly to prove, $\beta([z, x])G(y) = 0, \forall x, y, z \in S$ and $G(S) \subseteq Z(S)$

REFERENCES:

[1] M. Bresar, and J. Vukuman, On some additive mappings in rings with innovation, Aequations Math.,38(1989), 178-185.
[2] M. Chandramouleeswaran and V. Thiruveni, On derivations of semirings, Advances in Algebra,3(2010),123-131.
[3] M.Chandramouleeswaran and N.Sugantha Meena, Reverse Derivation on semirings, International Journal of Pure and Applied Mathematics,2(2015),203-212.
[4] Merve Ozdemir and Neset Aydin, $(\alpha, \beta) -$ Reverse Derivations on Prime and Semiprime Rings, International Journal of Open problems Compt. Math.,3(2018),48-59.

ON (gg)*-CLOSED SETS AND GENERALIZED ω-CLOSED SETS IN TOPOLOGICAL SPACES

A.Benazir[1] and N.Kandaraj[2]

[1]Research Scholar , PG and Research Department of Mathematics,
Saiva Bhanu Kshatriya College (Affiliated to Madurai Kamaraj University, Madurai),
Aruppukottai, Tamil Nadu, India. E.mail:benazirbenz@gmail.com

[2] Associate Professor, PG and Research Department of Mathematics,
Saiva Bhanu Kshatriya College (Affiliated to Madurai Kamaraj University, Madurai),
Aruppukottai,Tamil Nadu, India. Email:n.kandarajsbkc1998@gmail.com

ABSTRACT: *In this paper, we introduce generalization of generalized star closed sets ((gg*)-closed sets) and generalized ω-closed sets in topological spaces .*

Keywords: *Closed sets, Generalized closed set, (gg*)-closed sets, gg-open, g ω -closed sets.*

1. INTRODUCTION:

Closed sets are basic objects in a topological space. In 1970, N. Levine [3] initiated the study of g-closed sets . By Definition, a subset S of a topological space X is called generalized closed if $clA \subseteq U$ whenever $A \subseteq U$ and U is open. Generalized closed sets also proffer new properties of topological spaces and mainly are separation axioms weaker than T_1.In [1], Aull and Thron introduce several separation axioms between T_0 and T_1.Furthermore, the study of generalized closed sets also provide new characterization of some known classes of spaces for example the class of extremely disconnected spaces. Other new properties are defined by variations of the property of submaximality. In Section 2 , we follow a similar line to introduce generalized ω- closed sets by utilizing the ω-closure operator. We study g-closed sets and $g\omega$-closed setsin the spaces (X,τ) and (X,τ_ω). In particular, we show that a subset A of a space $(X,\tau)$is closed in (X,τ_ω) if and only if it is g-closed in (X,τ_ω) if and only if it is $g\omega$-closed in(X,τ_ω).

2.PRELIMINARIES

Throughout this paper (X,τ) denotes the topological space with no separation properties assumed.For a subset A of X , the closure of A and interior of A are denoted by $cl(A)$ and $int(A)$ respectively. A subset A of a topological space X is called *α-open* [resp. semi-open, preopen,semi-preopen] if $A \subseteq int(cl(int\ A))$ [resp. $A \subseteq cl(int\ A)$, $A \subseteq int(cl\ A)$, $A \subseteq cl(int(cl\ A))$]. Moreover, A is said to be *α-closed* [resp. semi-closed, preclosed,semi-preclosed] if X / A is α-open [resp. semi-open, preopen, semi-preopen] or,equivalently, if $cl(int(cl\ A)) \subseteq A$ [resp. $int(cl\,A) \subseteq A$, $cl(int\ A) \subseteq A$, $int(cl(int\,A)) \subseteq A$].

Let (X, τ) be a topological space and let A be a subset of X. The closure of A, the interior of A, and the relative topology on A will be denoted by $cl_{\tau}(A)$ (A), $\text{int}_{\tau}(A)$, and τ_A, respectively. The ω-interior (ω-closure) of a subset A of a space (X, τ) is the interior (closure) of A in the space (X, τ_{ω}), and is denoted by $\text{int}_{\tau\omega}(A)$ ($cl_{\tau_{\omega}}(A)$).

Definition 2.1. A space (X, τ) is called

(a) locally countable [4] if each point $x \in X$ has a countable open neighborhood;

(b) anti-locally countable [2] if each nonempty open set is uncountable;

(c) $T_{1/2}$-space [10] if every g-closed set is closed (equivalently if every singleton is open or closed, see [30]).

Definition 2.2. A function $f : (X, \tau) \to (Y, \sigma)$ is called

(a) g-continuous [5] if $f^{-1}(V)$ is g-closed in (X, τ) for every closed set V of (Y, σ);

(b) g-irresolute [5] if $f^{-1}(V)$ is g-closed in (X, τ) for every g-closed set V of (Y, σ);

(c) ω-continuous [11] if $f^{-1}(V)$ is ω-open in (X, τ) for every open set V of (Y, σ);

(d) ω-irresolute [12] if $f^{-1}(V)$ is ω-open in (X, τ) for every ω-open set V of (Y, σ);

(e) α-continuous [31] if $f^{-1}(V)$ is α-set in (X, τ) for every open set V of (Y, σ).

Lemma 2.3 .[4] *Let A be a subset of a space* (X, τ). *Then,*

(a)$(\tau_{\omega})_{\omega} = \tau_{\omega}$; (b) $(\tau_A)_{\omega} = (\tau_{\omega})_A$.

Definition 2.4.

(1) generalized closed set (g-closed) [3] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .

(2) Semi generalized closed [6] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is Semi open in X .

(3) generalized semi closed [8] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .

(4) generalized α -closed (gα -closed)[7] if $\alpha - cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α -open in X .

(5) α generalized closed (α g-closed) [9] if $\alpha - cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .

(6) generalized semi pre closed (gsp-closed)[13] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .

(7) generalized pre closed (gp-closed)[14] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .

(8) regular semi-open[15] if there is a regular open set U such that $U \subseteq A \subseteq cl(U)$.

(9) regular open set[16] if $A = \text{int}(cl(A))$.

(10) regular closed set if $A = cl(\text{int}(A))$.

(11) t-set [17] iff $\text{int}(A) = \text{int}(cl(A))$.

(12) regular generalized closed set (rg-closed)[18]if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X .

(13)generalized pre-regular closed (gpr-closed)[19] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X .

(14) generalized semi-pre regular closed (gspr-closed)[20]if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X .

(15) generalized star pre closed (g*p-closed)[21]if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open open in X .
(16) regular generalized α -closed($rg\alpha$ -closed)[22]if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular α -open in X .
(17)generalized α -closed($g\alpha$ -closed)[23]if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α -open in X .
(18) generalization of generalized closed set (gg-closed)[24]if $gcl(A) \subseteq U$ whenever and U is regular semi open in X .
(19) A topological space X is said to be locally indiscrete if every open subset is closed.
(20) R*-closed set [25] if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi open in X .

Definition 2.5. [27] A space X is said to be submaximal if every dense subset of X is open.A Space X is α -sub maximality (resp. g-submaximal, sg-submaximal) if every dense subset is α-open (resp g-open,sg-open)[26]. Obviously every submaximal space is g-submaximal, that if ($X,\alpha(X)$) is g-submaximal, then ($X,\alpha(X)$) is also sg-submaxima.

Remark 2.6. [28].Every semi-preclosed set is sg-closed and every preclosed set is g α -closed.

Definition 2.7. Let S be a subset of a space X .A resolution of S is a pair $< E_1, E_2 >$ of disjoint dense subsets of S. The subset S is said to be resolvable if it possesses a resolution, otherwise S is said to be irresolvable.

Definition 2.8. Let S ne a subset of a space X ,then S is called strongly irresolvable ,if every open subspace of S is irresolvable.

Remark 2.9. If $< E_1, E_2 >$ is a resolution of S then E_1 and E_2 are condense in X .i.e. have empty interior.

Lemma 2.10 [29] Every space X has a unique decomposition $X = F \cup G$ where F is closed and resolvable and G is open and hereditarily irresolvable. This decomposition is called Hewitt decomposition of X .

Theorem 2.11. [32] For a space X with Hewitt decomposition $X = F \cup G$ .Then the following are equivalent.
(1) every semi-preclosed subset of is X is sg-closed set.
(2) $X_1 \cap sclA \subseteq spclA$ for each $A \subseteq X_1$
(3). $X_1 \subseteq \text{int}(clG)$
(4) $X \approx Y \oplus Z$,where is locally indiscrete and Z is strongly irresolvable.
(5) every preclosed subset of X is g α -closed
(6) X is g-submaximal with respect to $\alpha(X)$.

3.(gg)*-CLOSED SETS

Definition 3.1. A subset A of a topological space (X,τ) is called generalization of generalized star closed sets (gg)*-closed if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is gg-open.

Proposition 3.2. Every regular closed set is (gg)*-closed.
Proof: Let A be a regular closed set in X such that $A \subseteq U$ and U is gg-open.

Then $rcl(A) \subseteq U$ .Therefore A is (gg)*-closed.

Proposition 3.3. Every (gg)*-closed set is g-closed.

Proof: Let A be a (gg)*-closed set in X .Let U be an open set in X such that $A \subseteq U$
Since every open set is gg-open[24] and since A is (gg)*-closed , $rcl(A) \subseteq U$.
But we have $cl(A) \subseteq rcl(A) \subseteq U$ Hence A is g-closed

Proposition 3.4. Every (gg)*closed set is gsp-closed

Proof: Let A be a (gg)*-closed set in X .Let U be a an open set in X such that $A \subseteq U$
Since every open set is gg-open[24] and since A is (gg)*-closed , $rcl(A) \subseteq U$.
But we have $spcl(A) \subseteq rcl(A) \subseteq U$ Hence A is gsp-closed.

Proposition 3.5. Every (gg)*closed set is gp-closed.

Proof: Let A be a (gg)*-closed set in X .Let U be an open set in X such that $A \subseteq U$
Since every open set is gg-open[24] and since A is (gg)*-closed , $rcl(A) \subseteq U$.
But we have $pcl(A) \subseteq rcl(A) \subseteq U$ Hence A is gp-closed.

Proposition 3.6. Every (gg)*closed set is gs-closed.

Proof : Let A be a (gg)*-closed set in X .Let U be an open set in X such that $A \subseteq U$
Since every open set is gg-open[24] and since A is (gg)*-closed $rcl(A) \subseteq U$.
But we have $scl(A) \subseteq rcl(A) \subseteq U$ .Hence A is gs-closed.

Proposition 3.7. Every (gg)*closed set is αg -closed.

Proof: Let A be a (gg)*-closed set in X .Let U be an open set in X such that $A \subseteq U$
Since every open set is gg-open[24] and since A is (gg)*-closed $rcl(A) \subseteq U$.
But we have $\alpha cl(A) \subseteq rcl(A) \subseteq U$ .Hence A is αg -closed.

Proposition 3.8. Every (gg)*closed set is rg-closed.

Proof: Let A be a (gg)*-closed set in X .Let U be an open set in X such that $A \subseteq U$
Since every regular open set is gg-open[24] and since A is (gg)*-closed , $rcl(A) \subseteq U$.
But we have $cl(A) \subseteq rcl(A) \subseteq U$ .Hence A is rg-closed.

Proposition 3.9. Every (gg)*closed set is gpr-closed.

Proof: Let A be a (gg)*-closed set in X .Let U be an open set in X such that $A \subseteq U$
Since every regular open set is gg-open[24] and A is (gg)*-closed , $rcl(A) \subseteq U$.
But we have $pcl(A) \subseteq rcl(A) \subseteq U$.
Hence A is gpr-closed.

Proposition 3.10. Every (gg)*closed set is gspr-closed.

Proof: Let A be a (gg)*-closed set in X .Let U be a regular open set in X such that $A \subseteq U$.
Since every regular open set is gg-open[24] and since A is (gg)*-closed , $rcl(A) \subseteq U$.
But we have $spcl(A) \subseteq rcl(A) \subseteq U$
Hence A is gspr-closed.

Proposition 3.11. Every (gg)*closed set is g*p-closed.

Proof: Let A be a (gg)*-closed set in X .Let U be a regular open set in X such that $A \subseteq U$.
Since every g- open set is gg-open[24] and since A is (gg)*-closed , $rcl(A) \subseteq U$ But we

have $pcl(A) \subseteq rcl(A) \subseteq U$

Hence A is g*p-closed.

Proposition 3.12. Every (gg)*closed set is g**-closed.

Proof: Let A be a (gg)*-closed set in X .Let U be a g*- open set in X such that $A \subseteq U$

Since every g*- open set is gg-open[24] and since A is (gg)*-closed , $rcl(A) \subseteq U$.

But we have $cl(A) \subseteq rcl(A) \subseteq U$.

Hence A is g**-closed.

4. GENERALIZED ω-CLOSED SETS :

Definition 4.1. A subset A of a space (X, τ) is called generalized ω-closed (briefly, $g\omega$-closed) if $cl_{\tau_\omega}(A) \subseteq U$ whenever $U \in \tau$ and $A \subseteq U$

We denote the family of all generalized ω-closed (generalized closed) subsets of a space (X, τ) by $G\omega C(X,\tau)(GC(X,\tau))$.

It is clear that if (X, τ) is a countable space, then $G\omega C(X,\tau) = P(X)$, where $P(X)$ is the power set of X .

Proposition 4.2. Every g-closed set is gω-closed.

The proof follows immediately from the definitions and the fact that τ_ω is finer than τ for any space (X, τ). However, the converse is not true in general as the following example shows.

Example 4.3. Let $X = \{a,b,c\}$ with the topology $\tau = \{\phi, X, \{a\}, \{a,b\}\}$ and let $A = \{a\}$.

Then $A \in G\omega C(X,\tau)$. But $A \notin GC(X,\tau)$ since $A \subseteq A \in \tau$ and $cl_\tau(A) = X \not\subset A$.

Lemma 4.4. Let (A, τ_A) be an anti-locally countable subspace of a space (X, τ).

Then $cl_\tau(A) = cl_{\tau\omega}(A)$.

Proof. We need to prove that $cl_\tau(A) \subseteq cl_{\tau_\omega}(A)$. Suppose that there exists $x \in cl_\tau(A) - cl_{\tau_\omega}(A)$. Then $x \notin cl_{\tau\omega}(A)$, and so there exists $W_x \in \tau_\omega$ such that $x \in W_x$ and $W_x \cap A = \Phi$ A is a nonempty countable open set in (A, τ_A)), which is a contradiction and the result follows.

Corollary 4.5. Let (A, τ_A) be an anti-locally countable subspace of a space (X, τ). Then $A \in GC(X,\tau)$ if and only if $A \in G\omega C(X,\tau)$.

Theorem 4.6. *Let* (X, τ) be any space and $A \subseteq X$. Then the following are equivalent.

1) A is ω-closed in (X, τ) (equivalently A is closed in$(, \tau_\omega)$).
2) $A \in GC(X, \tau_\omega)$
3) $A \in G\omega C(X, \tau_\omega)$

1)Proof. (a)⇒(b). It follows from the fact that every closed set is g-closed.

(b)⇒(c). It is obvious by using Proposition 4.2.

(c)⇒(a). We show that $cl_{\tau_\omega}(A) \subseteq A$. Suppose that $x_0 \notin A$ Then $U = X - \{x_0\}$ is an ω-open set containing A. Since $A \in G\omega C(X, \tau_\omega)$, $cl_{(\tau_\omega)\omega}(A) = cl_{\tau_\omega}(A) \subseteq U$, and thus $x_0 \notin cl_{\tau_\omega}(A)$. Therefore, $cl_{\tau_\omega}(A) \subseteq A$, that is, A is closed in (X, τ)

In the same way, it can be shown that a subset A of a space (X, τ) is closed if and only if $cl_\tau(A) \subseteq U$ whenever $U \in \tau_\omega$ and $A \subseteq U$.

Proposition 4.7. If $A \in GC(X, \tau_\omega)$, then $A \in G\omega C(X, \tau)$ but not conversely.

Example 4.8. Let $X = \mathbf{R}$ be the set of all real numbers with the topology $\tau = \{\phi, X, \{1\}\}$ and put $A =$ **R-Q**. Then A is an ω-open subset of (X, τ) such that $cl_{\tau_\omega}(A) =$ R-{1} $\not\subset A$ (i.e., $A \notin GC(X, \tau_\omega)$. However, $A \in G\omega C(X, \tau)$ since the only open set in (X, τ) containing A is X.

In Example 4.8, for a space (X, τ) the collections $GC(X, \tau)$ and $GC(X, \tau_\omega)$ are independent from each other.

Example 4.9. Conside $X = \mathrm{R}$ with the usual topology τ_u. Put $A = (0,1) \cap Q$ Then $cl_{(\tau_u)\omega}(A) = A$ (A is countable) ,and so $A \in \mathrm{GC}(\mathrm{R}, (\tau_u)_\omega)$. On the other hand, $A \notin GC$ (R, τ_u)since $U = (0,1)$ is open in (R, τ_u) such that $A \subseteq U$ and $cl_{\tau_u}(A) = [0,1] \not\subset U$.

In Example 4.9, (R, τ_u) is anti-locally countable and $A = (0,1) \cap Q \in G\omega C$ (R, τ_u) $- GC$ (R, τ_u). Thus the condition that (A, τ_A) is anti-locally countable in Corollary 4.5 cannot be replaced by the condition that (X, τ) is anti-locally countable.

Theorem 4.10. Let (X, τ) be an anti-locally countable space. Then (X, τ) is a T_1-space if and only if every $g\omega$-closed set is ω-closed.

Proof. We need to show the sufficiency part only. Let $x \in X$ and suppose that $\{x\}$ is not closed. Then $A = X - \{x\}$ is not open, and thus A is $g\omega$-closed (the only open set containing A is X). Therefore, by assumption, A is ω-closed, and thus $\{x\}$ is ω-open. So there exists $U \in \tau$ such that $x \in U$ and $U - \{x\}$ is countable. It follows that U is a nonempty countable open subset of (X, τ), a contradiction.

Proposition 4.11. *If* $A = \{A_\alpha : \alpha \in I\}$ is a locally finite collection of gω-closed sets of a space (X, τ), then $A = \bigcup_{\alpha \in I} A_\alpha$ is gω-closed (in particular, a finite union of gω-closed sets is gω-closed).

Proof. Let U be an open subset of (X, τ) such that A ⊆ U. Since $A_\alpha \in G\omega C(X, \tau)$ and $A_\alpha \subseteq U$ for each α ∈ I, $cl_{\tau_\omega}(A_\alpha) \subseteq U$. As τ_ω is a topology on X finer than τ, A is locally finite in (X, τ_ω). Therefore, $cl_{\tau_\omega}(A) = cl_{\tau_\omega}(\bigcup_{\alpha \in I} A_\alpha) = \bigcup_{\alpha \in I} cl_{\tau_\omega}(A_\alpha) \subseteq$ U. Thus, A is gω-closed in (X, τ).

Proposition 4.12. If A ∈ GωC(X, τ) and B is closed in (X, τ), then A ∩ B ∈ GωC(X, τ).

Proof: Let U be an open set in (X,τ) such that $A \cap B \subseteq U$. Put $W = X - B$. Then $A \subseteq U \cup W \in \tau$. Since $A \in G\omega C(X,\tau)$, $cl_{\tau_\omega}(A) \subseteq U \cup W$. Now, $cl_{\tau_\omega}(A \cap B) \subseteq cl_{\tau_\omega}(A) \cap cl_{\tau_\omega}(B) \subseteq cl_{\tau_\omega}(A) \cap cl_{\tau}(B) = cl_{\tau_\omega}(A) \cap B \subseteq (U \cup W) \cap B \subseteq U$.

Lemma 4.13. (a) If A is an ω-open subset of a space (X, τ), then $A - C$ is ω- open for every countable subset C of X.

(b) The open image of an ω-open set is ω-open.

Proof. Part (a) is clear. To prove part (b), let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an open function and let W be an ω-open subset of (X,τ). Let $y \in f(W)$. There exists $x \in W$ such that $y = f(x)$. Choose $U \in \tau$ such that $x \in U$ and $U - W = C$ is countable. Since f is open, $f(U)$ is open in (Y, σ) such that $y = f(x) \in f(U)$ and $f(U) - f(W) \subseteq f(U - W) = f(C)$ is countable. Therefore, $f(W)$ is ω-open in (Y, σ).

Theorem 4.14. Let (X, τ) and (Y, σ) be two topological spaces. Then $(\tau \times \sigma)_\omega \subseteq \tau_\omega \times \sigma_\omega$. Proof: Let $W \in (\tau \times \sigma)_\omega$ and $(x, y) \in W$. There exist $U \in \tau$ and $V \in \sigma$ such that $(x, y) \in U \times V$ and $U \times V - W = C$ is countable. Put $W_1 = (U \cap p_X(W)) - (p_X(C) - \{x\})$ and $W_2 = (V \cap p_Y(W)) - (p_Y(C) - \{y\})$, where $p_X : (X \times Y, \tau \times \sigma) \rightarrow (X, \tau)$ and $p_Y : (X \times Y, \tau \times \sigma) \rightarrow (Y, \sigma)$ are the natural projections. Then $W_1 \in \tau_\omega$, $W_2 \in \sigma_\omega$ (Lemma 4.13) and $(x,y) \in W_1 \times W_2 \subseteq W$. Thus $W \in \tau_\omega \times \sigma_\omega$.

Definition 4.15. A subset A of a space (X, τ) is called generalized ω-open (briefly, gω-open) if its complement $X - A$ is gω-closed in (X,τ).

It is clear that a subset A of a space (X,τ) is gω-open if and only if F $int_{\tau_\omega}(A)$, whenever $F \subseteq A$ and F is closed in (X, τ).

Theorem 4.16. If $A \times B$ is a gω-open subset of $(X \times Y, \tau \times \sigma)$, then A is gω-open in (X, τ) and B is gω-open in (Y, σ).

Proof. Let F_A be a closed subset of (X, τ) and let F_B be a closed subset of (Y, σ) such that $F_A \subseteq A$ and $F_B \subseteq B$. Then $F_A \times F_B$ is closed in $(X \times Y, \tau \times \sigma)$ such that $F_A \times F_B \subseteq A \times B$. By assumption, $A \times B$ is gω-open in $(X \times Y, \tau \times \sigma)$, and so $F_A \times F_B \subseteq int_{(\tau\times\sigma)_\omega}(A \times B) \subseteq int_{\tau_\omega}(A) \times int_{\sigma_\omega}(B)$ by using Theorem 4.14. Therefore, $F_A \subseteq int_{\tau_\omega}(A)$ and $F_B \subseteq int_{\sigma_\omega}(A)$, and the result follows.

The converse of the above theorem need not be true in general.

Example 4.17. Let X = Y = R with the usual topology τ_u. Let $A = R - Q$ and $B = (0, 3)$. Then A and B are ω-open subsets of (R, τ_u), while $A \times B$ is not gω-open in $(R \times R, \tau_u \times \tau_u)$, since $int_{(\tau_u \times \tau_u)_\omega} A \times B = \phi$ and $\{\sqrt{2}\} \times [1, 2]$ is a closed set in $(r \times r, \tau_u \times \tau_u)$ contained in $A \times B$.

Theorem 4.18. Let (Y, τ_Y) be a subspace of a space (X, τ) and $A \subseteq Y$. Then the following hold.

(a) If $A \in G\omega C(X, \tau)$, then $A \in G\omega C(Y, \tau_Y)$.

(b) If $A \in G\omega C(Y, \tau_Y)$ and Y is ω-closed in $(X \times Y, \tau)$, then $A \in G\omega C(X, \tau)$.

Proof. (a) Let V be an open set of (Y , τ_Y) such that A ⊆ V . Then there exists an open set U ⊆ τ such that V = Y ∩ U. Since A GωC(X, τ) and A U, cl_{τ_ω} (A) ⊆ U. Now, $cl_{(\tau_Y)_\omega}$ (A) = $cl_{(\tau_\omega)Y}(A)$ = cl_{τ_ω} (A) ∩ Y ⊆ Y ∩U = V . Therefore, A ∈ GωC(Y , τ_Y).

(b) Let A ⊆ U, where U ∈ τ. Then A ⊆ Y ∈ U ∈ τ_Y . Since A ∈ GωC(Y , τ_Y), $cl_{(\tau_Y)_\omega}$ (A)= $cl_{(\tau_\omega)_Y}$ (A) = $cl_{(\tau_\omega)}$ (A) ∩ Y ⊆Y ∩U. Finally, cl_{τ_ω} (A) = cl_{τ_ω} (A ∩ Y) ⊆ cl_{τ_ω} (A) ∩ cl_{τ_ω} (Y) = (Y is ω-closed) cl_{τ_ω} (A) ∩ Y ⊆ Y ∩ U ⊆ U. Thus A ∈ GωC(X, τ).

REFERENCES:

[1] C.E.Aull and W.J.Thron,separation axioms between T_0 and T_1 ,Indag Math.,23(1962),26-37.

[2] S.Al –Ghour, Certain Covering Properties related to Paracompactness, Ph.D.thesis, University of Jordan,Amman,1999.

[3] N.Levine ,Generalized closed sets in topological spaces, Rend circ. Mat. Palermo, 19 (1970), 89-96.

[4] K.Al-Zoubi and B.Al-Nashef, The topology of ω -open subsets, Al-Manarah 9 (2003) No.2, pp. 169-179.

[5] K. Balachandran, P.Sundaram,and H.Maki, On generalized continuous maps in topological spaces, Mem.Fac.Sci.Kochi Univ. Ser. A Math.**12** (1991),5-13.

[6] P.Bhattacharya and B.K.Lahiri, Semi-generalized closed sets in topology, Indian. J. Math, 29(1987),375-382.

[7] H. Maki, R. Devi and K. Balachandran, Generalized α-closed sets in topology, Bull. Fukuoka Univ. Ed. Part III, 42 (1993), 13-21.

[8] S.Arya and T.Nour, Characterizations of s -normal spaces, Indian J. Pure Appl. Math. 21(1990), 717-719.

[9] H. Maki, K. Balachandran and R. Devi, Associated topologies of generalized α-closed sets and α-generalized closed sets, Mem. Fac. Sci. Kochi Univ. Ser. A, Math., 15 (1994), 51-63.

[10] K. Al-Zoubi, Semi ω -continuous functions, Abhath Al-Yarmouk 12(2003). No.1,119-131.

SELF MAPS ON QS-ALGEBRA

P. Jainumbu Beevi

Assistant Professor, PG and Research Department of Mathematics

Saiva Bhanu Kshatriya College

Aruppukottai -626101, Tamil Nadu, India

Abstract: *In this paper, we define the concept of Self maps(Left & Right maps) on QS-Algebra and investigate some of their elegant and simple results.*

AMS Classification: *06F35, 03G25*

Key words: *QS-algebras, Left(L)- maps and Right(R)-maps on QS-algebras.*

1. INTRODUCTION

Ahn and Kim [1] proposed the notion of QS-algebras which also a generalization of BCK/BCI-algebras. In [2], Y.B. Jun, E.H. Kim introduced a new class of algebras, called BH-algebras, which is also generalization of BCH/BCI/BCI-algebras. In [4], the authors, studied some relations between Left-(Right-) maps and positive implicativity in BH-algebras. We introduced some special type of mapping on QS-algebras called the left (or) right maps in QS-algebras X.

2. PRELIMINARIES

In this section, we recall some basic definition and results that are required for our work.

Definition 2.1: [1] A QS-algebras $(X,*,0)$ is a non-empty X with the constant 0 and single binary operation * satisfying the following actions:

1. $x * x = 0$
2. $x * 0 = x$
3. $(x * y) * z = (x * z) * y$
4. $(x * y) * (x * z) = z * y$ *for all x, y, z in X*

Example 2.2: Let (X={0,1,2,3}, * , 0) be a set with the following Cayley table:

*	0	1	2	3
0	0	1	2	3
1	1	0	1	1
2	2	1	0	0
3	3	1	3	0

Then (X,*,0) is QS-algebras.

Remark 2.3:[2] Let $(X,*,0)$ be a QS-algebras. Define $x \wedge y = y * (y * x)$, *for all x, y in X.*

A QS-algebras X is said to be commutative if $x \wedge y = y \wedge x$, *for all x, y in X.*

Definition 2.4:[3] A BH-algebras $(X,*,0)$ is a non empty set X with a constant 0 and single binary operation * satisfying the following axioms:

1. $x * x = 0$
2. $(x * y) * z = (x * z) * y$
3. $x * y = 0$ *and* $y * x = 0 \Rightarrow x = y$, *for all x, y, z in X.*

Example 2.5: Let (X={0,1,2,3}, * , 0) be a set with the following Cayley table:

*	0	1	2	3
0	0	0	0	0
1	1	0	3	1
2	2	2	0	3
3	3	1	2	0

Then (X,*,0) is BH-algebras.

Definition 2.6: [4] Let *(X,*,0)* be BH-algebras.

- For fixed a in X, we define a map $R_a : X \rightarrow X$ such that $R_a(x) = x * a, \ \forall x \in X$. Then R_a is called a right map on X. The set of all right map on X is denoted by **R** .
- For fixed a in X, we define a map $L_a : X \rightarrow X$ such that $L_a(x) = a * x, \ \forall x \in X$. Then L_a is called a right map on X. The set of all left map on X is denoted by **L**.

Example 2.7: Let (X={0,1,2}, * , 0) be a BH-algebras with following Cayley table:

*	0	1	2
0	0	0	0
1	1	0	1
2	2	2	0

Define a function $f : X \rightarrow X$ by $f(x) = 0$ if $x = 0,1$ and $f(x) = 2$ if $x=2$
Fix $a= 2$, the map $R_2(x)= x * 2$, *for all* x *in* X. Hence the function f on X becomes a right map R_2 on X

Example 2.7: Let (X={0,1,2}, * , 0) be a BH-algebras with following Cayley table:

*	0	1	2
0	0	0	0
1	1	0	1
2	2	2	0

Define a function $f : X \rightarrow X$ by $f(x) = 1$ if $x = 0,2$ and $f(x) = 0$ if $x=1$
Fix $a= 1$, the map $L_2(x)= 1 * x, \ \forall x \in X$. Hence the function f on X becomes a left map L_2 on X.

Definition 2.8: [1] A subset A of a QS-algebras X is called and ***ideal*** of X if it satisfies:

1. $0 \in A$
2. *for all* $y \in A$ and $x * y \in A$ *imply* $x \in A$, *for all* $x \in X$.

   Obviously, $\{0\}$ and X are ideal of X.

Definition 2.9: [1] If *(X,*,0)* be a QS-algebras then we define a partial ordering $\leq$ by $x \leq y$ if $x * y=0$.

Definition 2.10: [4] Let X be a BH-algebras and let R_a and L_a be a right and left maps on X. We have the following subsets of X corresponding to L_a and R_a respectively.

Ker $(L_a) = \{x \in X / L_a(x) = 0\}$

Ker $(R_a) = \{x \in X / R_a(x) = 0\}$

3. LEFT AND RIGHT MAPS ON QS-ALGEBRAS

Definition 3.1: Let $(X,*,0)$ be QS-algebras.

- For fixed a in X, we define a map $L_a : X \rightarrow X$ such that $L_a(x) = a * x$, $\forall x \in X$. Then L_a is called a right map on X. The set of all left map on X is denoted by **L**.
- For fixed a in X, we define a map $R_a : X \rightarrow X$ such that $R_a(x) = x * a$, $\forall x \in X$. Then R_a is called a right map on X. The set of all right map on X is denoted by **R** .

Example 3.2: Let (X={0,1,2}, * , 0) be a QS-algebras with following Cayley table:

*	0	1	2
0	0	2	1
1	1	0	2
2	2	0	2

Define a function $f : X \rightarrow X$ by $f(x) = 2$ *if* $x = 0$ *and* $f(x) = 0$, *otherwise*

Fix $a= 1$, the map $R_1(x)= x * 1$, $\forall x \in X$. Hence the function f on X becomes a right map R_1 on X

Example 3.3: Let (X={0,a,b,c}, * , 0) be a set with the following Cayley table:

*	0	1	2	3
0	0	1	2	3
1	1	0	1	1
2	2	1	0	0
3	3	1	3	0

Define a function $f : X \rightarrow X$ by $f(x) = 0$ *if* $x=2,3$, $f(x)= 2$ *if* $x=0$ *and* $f(x)= 1$ *if* $x=1$.

Fix $a=2$, the map $L_2(x)= 2 * x$, $\forall x \in X$. Hence the function f on X becomes a right map L_2 on X

Proposition 3.4: Let X be a QS-algebras. Then for any x, y *and* z *in* X, the following results holds:

1. $x * (x * y) = y$
2. $0 * (x * y) = y * x =(0 * x) *(0 * y)$
3. $(x * (x * y)) * y = 0$
4. *If* $x * y = 0$ *and* $y * x = 0$ *then* $x = y$, *for all* x, y *in* X
5. $(x * z) * (y * z) = x * y$

Proof:

1. $(x * y) = (x * 0) * (x * y)$ *by (1) of definition 2.1*
   $= y * 0$ *by (4) of definition 2.1*
   $= y$ *by (1) of definition 2.1*
2. $0 * (x * y) = (x * x) * (x * y)$ *by (1) of definition 2.1*
   $= y * x$ *by (4) of definition 2.1*
   $= (0 * x) *(0 * y)$ *by (4) of definition 2.1*
3. $(x * (x * y)) * y = ((x * 0) * (x * y)) * y$ *by (2) of definition 2.1*
   $= (y * 0) * y$ *by (4) of definition 2.1*
   $= y * y$ *by (2) of definition 2.1*
   $= 0$ *by (1) of definition 2.1*

4. $x = x * 0 = x * (x * y) = (x * 0) * (x * y) = y * 0 = y$
   $y = y * 0 = y * (y * x) = (y * 0) * (y * x) = x * 0 = x$
5. *Suppose* $(x * z) * (y * z) \neq x * y$. *Then*

$$((x * z) * (y * z)) * (x * y) \neq (x * y) * (x * y)$$
$$\neq y * y \quad \text{by (4) of definition 2.1}$$
$$\neq 0 \quad \text{by (1) of definition 2.1}$$

This contradiction the condition $((x * z) * (y * z)) * (x * y) =0$, and prove that $(x * z) * (y * z) = x * y$

Proposition 3.5 Let X be a QS-algebras.
For every natural number n, $L_a{}^n = L_a$ *if* n *odd and* $L_a{}^n = L_a{}^2$ *if* n *is even.*
Proof: Let $x \in X$.

Let $x \in X$.

Case (i): *n is odd.*

Assume now that, the result is true for $n=2m+1$.

That is, $L_a{}^{2m+1}(x)= L_a(x)$ .... *(1)*

Now, $L_a{}^{2m+3}(x)= L_a{}^2(L_a{}^{2m+1}(x))= L_a{}^2(L_a(x))$ *by (1)*

$$= L_a{}^3(x)$$
$$= L_a(x)$$

Thus the result is true for any n which an odd number.

Case (ii): *n is even.*

Again assume that, the result is true for $n=2m$.

That is, $L_a{}^{2m}(x)= L_a{}^2(x)$ .... *(2)*

Now, $L_a{}^{2m+2}(x)= L_a{}^2(L_a{}^{2m}(x))= L_a{}^2(L_a{}^2(x))$ *by (2)*

$$= L_a{}^4(x)$$
$$= L_a{}^2(x)$$

Thus the result is true for any n which an even number.

Hence the result is true for every natural number n, $L_a{}^n = L_a$ *if* n *odd and* $L_a{}^n = L_a{}^2$ *if* n *is even.*

Proposition 3.6: Let X be a QS-algebras. Then for all x, y *in* X, we have
1. $L_a{}^2(x) * L_a(y) = L_a{}^2(y) * L_a(x)$
2. $L_a{}^2(x) * y = L_a(y) * L_a(x) = L_a{}^2(x) * L_a{}^2(y)$

Proof: Let $x, y \in X$.
1. $L_a{}^2(x) * L_a(y) = (a * (a * x)) * (a * y)$ *by definition 3.1*
   $= (a * (a * y)) * (a * x)$ *by (3) of definition 2.1*
   $= L_a{}^2(y) * L_a(x)$ *by definition 3.1*
2. $L_a{}^2(x) * y = (a * (a * x)) * y$
   $= (a * y) * (a * x)$ *by (3) of definition 2.1*
   $= L_a(y) * L_a(x)$ ....*(1')* *by definition 3.1*

   $L_a{}^2(x) * L_a{}^2(y) = (a * (a * x)) * (a * (a * y))$
   $= (a * (a * (a * y))) * (a * x)$ *by (3) of definition 2.1*
   $= (a * y) * (a * x)$ *by proposition 3.4*
   $= L_a(y) * L_a(x)$ *by definition 3.1* ………..*(2')*

From (1') and (2'), we get $L_a^2(x) * y = L_a(y) * L_a(x) = L_a^2(x) * L_a^2(y)$

Proposition 3.7: Let X be a QS-algebras. Then the following results hold,

1. *L_a^2 is isotonic, i.e, $x \leq y$ implies $L_a^2(x) \leq L_a^2(y)$*
2. *$L_a^2(x) = 0$ if and only if $R_x(a) = a$*

Proof: Let $x \in X$.
Let $x \leq y$. Then $x * y = 0$ ……… (1') *by definition 2.8*

1. $L_a^2(x) * L_a^2(y) = L_a^2(x) * y$ *by proposition 3.6*
   $= (a * (a * x)) * y$
   $= x * y$ *by (1) of proposition 3.4*
   $= 0$ *by (1')*

   By definition of partial order, we get $L_a^2(x) \leq L_a^2(y)$
2. Let $L_a^2(x) = 0$ *iff* $L_a(L_a(x)) = 0$
   iff $a * (a * x) = 0$ *by definition 3.1*
   iff $a * (a * x) = a * a$ *by (1) of definition 2.1*
   iff $a * R_x(a) = a * a$ *by definition 3.1*
   iff $R_x(a) = a$ *by left cancellation law*

Proposition 3.8: Let X be a QS-algebras and let L_a be a left map on X. If $x \in Ker(L_a)$ and $y \in X$, then $x \wedge y \in Ker(L_a)$.
Proof: Let $y \in X$
Let $x \in Ker(L_a)$. Then $L_a(x) = 0$ ……… (1) *by definition 2.10*

$L_a(x \wedge y) = L_a(y * (y * x))$ *by remark 2.3*
$= L_a(x)$ *by proposition 3.4*
$= 0$ *by (1)*

Therefore $x \wedge y \in Ker(L_a)$.

Proposition 3.9: Let X be a QS-algebras. Then for any a in X, $Ker(L_a^2)$ is ideal of X.
Proof: Since $L_a^2(0)=0$, we get $0 \in Ker(L_a^2)$
If $y, x * y \in Ker(L_a^2)$ then $L_a^2(y)=0$ and $L_a^2(x * y)=0$ ……… (1)

$L_a^2(x) = L_a^2(x) * L_a^2(x * y)$ *by (1)*
$= L_a^2(x) * (x * y)$ *by (2) of proposition 3.6*
$= (L_a^2(x) * L_a^2(y)) * (x * y)$ *by (1)*
$= (L_a^2(x) * y) * (x * y)$ *by (2) of proposition 3.6*
$= L_a^2(x) * x$ *by (5) of proposition 3.4*
$= L_a(x) * L_a(x)$ *by (2) of proposition 3.6*
$= 0$ *by (1) definition 2.1*

Therefore, $x * y \in Ker(L_a^2)$. Hence $Ker(L_a^2)$ is *ideal* of X.

REFERENCES:

[1] Sun Shin Ahn and Hee Sik Kim, On QS-algebras, Journal of the Chungcheong Mathematical Society, volume 12 August 1999, 33-41

[2] Michiro Kondo, On the class of QS-algebras, Hindawi Publishing Corporation International Journal of Mathematics and Mathematical Science, 2004:49, 2629-2639.

[3] Y.B. Jun, E.H. Roh and H.S. Kim, On BH- algebras, Sci. Math. 1(1998), 347-354.
[4] Sun Shin Ahn and Hee Sik Kim, R- maps and L- maps in BH-algebras, Journal of the Chungcheong Mathematical Society, volume 13(2) December 2000, 53-59.

PYTHAGOREAN FUZZY ON β – ALGEBRAS

K. Sujatha

Assistant Professor,
Saiva Bhanu Kshatriya College,
Aruppukottai - 626101.Tamil Nadu. India.
ksujatha203@gmail.com

ABSTRACT: *In this paper, introduce the notion on Pythagorean fuzzification of subalgebra sin β – algebra. Also discuss some basic concepts of results and investigate their several properties.*

Keywords: *β - algebra, Pythagorean fuzzy set, Pythagorean fuzzy β - algebra, Pythagorean fuzzy level set*

AMS Classification: *08A72, 03E72*

1. INTRODUCTION:

In 1965, Fuzzy set (FS) was developed by Lofti.A Zadeh [5] and he discussed membership function only. By this way, in 1986, Atanasov [1] introduced the notion of Intuitionistic Fuzzy set (IFS) in which not only the membership value is considered but also consider non-membership values. After that, many researchers used the fuzzy and Intuitionistic Fuzzy set apply in many areas. In another extended of a Fuzzy set, in 2013, Yager [3] [4], introduced a new concept of non – standard fuzzy sets called a Pythagorean fuzzy sets (PFS) and related ideas of Pythagorean membership function grades. In 2002, J. Neggers and H.S. Kim [2], introduced a class of algebras called β- algebras. This paper dealt the idea of Pythagorean fuzzy on β- sub algebras and Pythagorean fuzzy on level β- algebra, by connecting the concepts β-algebras, Pythagorean fuzzy set. Also proved some of their properties and relation between Intuitionistic Fuzzy β- algebras and Pythagorean fuzzy β-algebras.

2. PRELIMINARIES:

In this section we recall some basic definitions that are required in the sequel.

Definition 2.1: A β-algebra is a non-empty set X with a constant 0 and two binary operations + and - satisfying the following axioms:

1. $x - 0 = x$
2. $(0 - x) + x = 0$
3. $(x - y) - z = x - (z + y)$ for all x, y, z ϵ X.

Definition 2.2: Let X be a set of universal discourse and a fuzzy set μ in X is a function $\mu : X \rightarrow [0, 1]$. For each element x in X, μ(x) lies between 0 and 1 and μ(x) is called the membership value of x in X.

Definition 2.3: A non-empty subset I of a β− algebra (X, +, −, 0) is called a β− ideal of X, if 1. $0 \in I$

2. $x + y \in I \;\forall\; x, y \in X$
3. if $x - y$ and $y \in I$ then $x \in I \;\forall\; x, y \in X$.

Definition 2.4: Let μ be a fuzzy set in a β− algebra X. Then μ is called a fuzzy β− subalgebra of X if

1. $\mu(x + y) \geq \min\{\mu(x), \mu(y)\}$ $\forall$ x, y ∈ X.
2. $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$ $\forall$ x, y ∈ X.

Definition 2.5: An intuitionistic fuzzy set in a nonempty set X is defined by $A = \{< x, \mu_A(x)(x), \nu_A(x) > / x \in X\}$, $\forall x \in X$, where $\mu_A : X \to [0, 1]$ is a membership function of A. $\nu_A : X \to [0, 1]$ is a non-membership function of A and satisfies $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Definition 2.6: Let (X, +, −, 0) be a β algebra. An Intuitionistic fuzzy set $A = \{x, \mu_A(x), \nu_A(x) \mid x \in X\}$ is called an Intuitionistic fuzzy (IF) β subalgebra of X, if it satisfies the following conditions.

1. $\mu_A(x + y) \geq \min(\mu_A(x), \mu_A(y))$ and $\nu_A(x + y) \leq \max(\nu_A(x), \nu_A(y))$,
2. $\mu_A(x - y) \geq \min(\mu_A(x), \mu_A(y))$ and $\nu_A(x - y) \leq \max(\nu_A(x), \nu_A(y))$, $\forall$ x, y ∈ X,
   where $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Definition 2.7: Pythagorean Fuzzy Set

Let X be a non-empty set. A Pythagorean fuzzy set 'A' is an object having the form $A = \{<x, \mu_A(x), \nu_A(x) > \mid x \in X\}$ $\forall x \in X$, where the membership function $\mu_A : X \to [0,1]$ and the non-membership function $\nu_A : X \to [0,1]$ respectively and satisfies $0 \leq \mu_A(x)^2 + \nu_A(x)^2) \leq 1$.

Definition 2.8: Let X and Y be two β- algebras. A mapping f : X → Y is said to be a β - homomorphism, if $f(x + y) = f(x) + f(y)$ and $f(x - y) = f(x) - f(y)$ for all x, y ϵ X.

3. PYTHAGOREAN FUZZY ON β – ALGEBRAS :

In this section, introduce the notion of Pythagorean fuzzy β- subalgebra on β- algebra. We begin with the definition and example. Also discuss relation between Intuitionistic fuzzy β- subalgebra and Pythagorean fuzzy β- subalgebra.

Definition 3.1:

Let (X, +, −, 0) be a β algebra. A Pythagorean fuzzy set $A = \{<x, \mu_A(x), \nu_A(x) > \mid x \in X\}$ is called a Pythagorean fuzzy (PF) β- subalgebra of X, if it satisfies the following conditions

(1) $\mu_A(x + y) \geq \min(\mu_A(x), \mu_A(y))$ and $\nu_A(x + y) \leq \max(\nu_A(x), \nu_A(y))$,

(2) $\mu_A(x - y) \geq \min(\mu_A(x), \mu_A(y))$ and $\nu_A(x - y) \leq \max(\nu_A(x), \nu_A(y))$, $\forall$ x, y ∈ X,

Where $0 \leq \mu_A(x)^2 + \nu_A(x)^2 \leq 1$.

Example 3.2: The β-algebra X = ({0, 1, 2}, +, -, 0) with the following Cayley's table.

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

-	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

A Pythagorean fuzzy β – subalgebra is defined by

$$\mu_A(x) = \begin{cases} 0.4, if\ x = 0 \\ 0.3,\ otherwise \end{cases} \quad \text{and} \quad \vartheta_A(x) = \{0.8, if\ x = 0,1,2,3$$

Then we can observe that A is a Pythagorean fuzzy β – subalgebra of X.

Example 3.3: Consider the β− algebra of example 3. 2. The set A defined by

$$\mu_A(x) = \begin{cases} 0.4, if\ x = 0 \\ 0.3\ ,\ otherwise \end{cases} \quad \text{and} \quad \vartheta_A(x) = \begin{cases} 0.7, if\ x = 0 \\ 0.8,\ \ if\ x = 1 \\ 0.2\ ,\ if\ x = 2 \end{cases}$$

is not a Pythagorean fuzzy β− subalgebra of X.
For, ϑ_A (1 + 1) ≥ max {ϑ_A (1), ϑ_A (1)} ⇒ ϑ_A (2) ≥ max {ϑ_A (1), ϑ_A (1)} ⇒ .2 ≥ max {.8, .8}.

Theorem 3.4 Let A and B be a Pythagorean fuzzy β− subalgebras of X. Then A ∩ B is also a Pythagorean fuzzy β− subalgebra of X.

In general, the intersection of a family of Pythagorean fuzzy β− subalgebras of X is also a Pythagorean fuzzy β− subalgebra of X.

Proposition 3.5: Every Pythagorean fuzzy β− subalgebra of X satisfies the following condition.

$\mu_A(0) \geq \mu_A(x)$ and $\vartheta_A(0) \leq \vartheta_A(x)$ for all x ∈ X.

Proof:

For any x ∈ X. μ_A (0) = μ_A (x − x) ≥ min (μ_A (x), μ_A (x)) = μ_A (x).
Therefore μ_A (0) ≥ μ_A (x).
And ϑ_A (0) = ϑ_A (x − x) ≤ max (ϑ_A (x), ϑ_A (x)) = ϑ_A (x).
Therefore ϑ_A (0) ≤ ϑ_A (x).

Theorem 3.6 If A is a Pythagorean fuzzy β− subalgebra of X, then μ_A (x) ≤ μ_A (x − 0) and ϑ_A (x) ≥ ϑ_A (x − 0).

Proof:

Let A be a Pythagorean fuzzy β− subalgebra of X.

$$\begin{aligned} \text{Then } \mu_A(x-0) &\geq \min(\mu_A(x), \mu_A(0)) \\ &= \min(\mu_A(x), \mu_A(x-x)) \\ &\geq \min\{(\mu_A(x), \min(\mu_A(x), \mu_A(x))\} \\ &= \min(\mu_A(x), \mu_A(x)) \\ &= \mu_A(x) \end{aligned}$$

Similarly, we can prove that, ϑ_A (x − 0) ≤ ϑ_A (x).

Definition 3.7: Let f : X → Y be a function. Let A and B be two Pythagorean β− subalgebras in X and Y respectively. Then inverse image of B under f is defined by f^{-1} (B) = { f^{-1} (μ_B (x)), f^{-1} (ϑ_B (x))|x ∈ X} such that f^{-1} (μ_B (x)) = (μ_B (f(x))) and f^{-1} (ϑ_B (x)) = (ϑ_B (f(x))).

Theorem 3.8: Let X and Y be two Pythagorean β− subalgebras. Let f : X → Y be a homomorphism. If A is of Pythagorean β− subalgebra of Y, then f^{-1} (A) is a Pythagorean β− subalgebra of X.

Proof:

Let A be a Pythagorean β− subalgebra of Y, x, y ∈ Y.

$$\begin{aligned} f^{-1}(\mu_A(x+y)) &= \mu_A(f(x+y)) \\ &= \mu_A(f(x)+f(y)) \geq \min\{\mu_A f(x)), \mu_A f(y))\} \\ &= \min(f^{-1}(\mu_A(x)), f^{-1}(\mu_A(y)) \end{aligned}$$

And f^{-1} (μ_A (x − y)) ≥ min { f^{-1} (μ_A (x))), f^{-1} (μ_A (y))}

Similarly, we can prove,

$f^{-1}(\vartheta_A(x + y)) = \vartheta_A(f(x + y))$

$= \vartheta_A(f(x) + f(y))$

$\leq \max\{\vartheta_A f(x)), \vartheta_A f(y))\}$

$= \max(f^{-1}(\vartheta_A(x)), f^{-1}(\vartheta_A(y))$

And $f^{-1}(\vartheta_A(x - y)) \leq \max\{f^{-1}(\vartheta_A(x)), f^{-1}(\vartheta_A(y))\}$

Hence $f^{-1}(A)$ is a Pythagorean β− subalgebra of X.

Relation between Pythagorean fuzzy β− subalgebra and Intuitionistic fuzzy β- subalgebra

Remark 3.9: Every Intuitionistic fuzzy β− subalgebra is a Pythagorean fuzzy β- subalgebra. But converse need not be true.

That means, Every Pythagorean fuzzy β- subalgebra is not an Intuitionistic fuzzy β− subalgebra.

The above example 3.2, Pythagorean fuzzy β- subalgebra is not an Intuitionistic fuzzy β− subalgebra.

For, an Intuitionistic fuzzy β− subalgebra satisfies, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

But x = 1, $(\mu_A(1) + \nu_A(1)) \Rightarrow (0.3 + 0.8) \Rightarrow 1.1 \notin [0,1]$.

Then we can observe that A is a Pythagorean fuzzy β – subalgebra of X but not an Intuitionistic fuzzy β− subalgebra.

Now, we define the Cartesian product of the two PF β− subalgebras A and B of the β− algebras X and Y respectively.

Definition 3.10: Let $A = \{<x, \mu_A(x), \nu_A(x)> \mid x \in X\}$ and $B = \{<y, \mu_B(y), \nu_B(y)> \mid x \in Y\}$ be two Intuitionistic fuzzy β− subalgebras of X and Y respectively. The Cartesian product of A and B is $A \times B = \{(\mu_A \times \mu_B)(x, y)$ and $(\nu_A \times \nu_B)(x, y) \mid x, y \in X \times Y\}$ where $(\mu_A \times \mu_B)(x, y)$ =

Min $(\mu_A(x), \mu_B(y))$ and $(\nu_A \times \nu_B)(x, y) = \max(\nu_A(x), \nu_B(y))$.

Theorem 3.11: Let A and B be PF β− subalgebras of X and Y respectively. Then A×B is a PF β− subalgebra of X ×Y.

Proof:

Take $x = (x_1, x_2), y = (y_1, y_2) \in X \times Y$, $\mu_{(A\times B)} = \mu_A \times \mu_B$ and $\nu_{(A\times B)} = \nu_A \times \nu_B$.

$\mu_{(A\times B)}(x + y) = \mu_{(A\times B)}((x_1, x_2) + (y_1, y_2))$

$= (\mu_A \times \mu_B)(x_1 + y_1), (x_2 + y_2))$

$= \min\{\mu_A(x_1 + y_1), \mu_B(x_2 + y_2)\}$

$\geq \min\{\min(\mu_A(x_1), \mu_A(y_1)), \min(\mu_B(x_2), \mu_B(y_2))\}$

$= \min\{\min(\mu_A(x_1), \mu_B(x_2)), \min(\mu_A(y_1), \mu_B(y_2))\}$

$= \min\{(\mu_A \times \mu_B)(x_1, x_2), (\mu_A \times \mu_B)(y_1, y_2)\}$

$= \min\{(\mu_A \times \mu_B)(x), (\mu_A \times \mu_B)(y)\}$

Similarly, $\mu_{(A\times B)}(x - y) \geq \min\{(\mu_A \times \mu_B)(x), (\mu_A \times \mu_B)(y)\}$.

Analogously, we can prove that,

$\nu_{(A\times B)}(x + y) \leq \max\{(\nu_A \times \nu_B)(x), (\nu_A \times \nu_B)(y)\}$ and

$\nu_{(A\times B)}(x - y) \leq \max\{(\nu_A \times \nu_B)(x), (\nu_A \times \nu_B)(y)\}$.

Theorem 3.12: Let A × B be a PF β− subalgebra of X × X.

Then the following hold

1. either $\mu_A(x) \leq \mu_A(0)$ or $\mu_B(x) \leq \mu_B(0)$

2. either ν_A (x) ≥ ν_B (0) or ν_B (x) ≥ ν_B (0)

Proof: 1) Suppose μ_A (x) > μ_A (0) and μ_B (x) > μ_B (0), for all x, y ∈ X.

Then $(\mu_A \times \mu_B)(x + y) \geq \min(\mu_A(x), \mu_A(y))$

$> \min(\mu_A(0), \mu_A(0))$

$= (\mu_A \times \mu_B)(0, 0)$

which is contradiction.

Similarly, $(\mu_A \times \mu_B)(x - y) > (\mu_A \times \mu_B)(0, 0)$. which is contradiction.

2) Proceeding as in part (1), we can prove (2).

4. LEVEL OF PF- β SUBALGEBRAS:

In this section, we introduce the notion of level subsets of PF β− subalgebras of β− algebra.

Definition 4.1:

Let A be PF- β subalgebra of X, s, t ∈ [0, 1]. Then $A_{s,t} = \{x \in X \mid \mu_A(x) \geq s, \nu_A(x) \leq t\}$ where

$0 \leq \mu_A(x)^2 + \nu_A(x)^2) \leq 1$ is called a level such that associated with the PF- β subalgebra of A. Clearly, $A_{s,t} \subseteq X$.

Theorem 4.2: If A = (μ_A, ν_A) is a PF β− subalgebra of X, then the set $A_{s,t}$ is a β− subalgebra of X, for every s, t ∈ [0, 1].

Proof: For x, y ∈ $(\mu_A)s$, then μ_A (x) ≥ s and μ_A (y) ≥ s

$\Rightarrow \mu_A(x + y) \geq \min\{\mu_A(x), \mu_A(y)\}$

$\geq \min\{s, s\}$

$\geq s$

$\Rightarrow x + y \in (\mu_A)s$

Similarly it can be proved that $x - y \in (\mu_A)s$

And $\nu_A(x + y) \leq \max\{\nu_A(x), \nu_A(y)\}$

$\leq \max\{t, t\}$

$\leq t$

$\Rightarrow x + y \in (\nu_A)t$

Similarly it can be proved that $x - y \in (\nu_A)t$

Hence $A_{s,t}$ is subalgebra of X.

Theorem 4.3: Let A = (μ_A, ν_A) is a PF set in X such that $A_{s,t}$ is a β− subalgebra of X for every s, t ∈ [0, 1]. Then A is a PF β− subalgebra of X.

Proof: Let A = (μ_A, ν_A) is a PF set in X.

Assume that $A_{s,t}$ is a β− subalgebra of X for every s, t ∈ [0, 1].

∴ $x + y \in A_{s,t} \Rightarrow \mu_A(x + y) \geq s$ and $\nu_A(x + y) \leq t$.

That is $\mu_A(x + y) \geq s = \min\{\mu_A(x), \mu_A(y)\}$ and $\mu_A(x + y) \leq t = \max\{\nu_A(x), \nu_A(y)\}$

Similarly, we can prove that $\mu_A(x-y) \geq s = \min\{\mu_A(x), \mu_A(y)\}$ and

$\mu_A(x-y) \leq t = \max\{\mu_A(x), \nu_A(y)\}$

Thus A = (μ_A, ν_A) is an IF β− subalgebra of X.

Theorem 4.4: Let A = (μ_A, ν_A) be an IF- β subalgebra of X iff for all s, t ∈ [0, 1] level set $A_{s,t}$ is either empty or β− subalgebra of X.

Proof: Straight forward.

5. CONCLUSION:

In this paper several interesting results were discussed by joining the notions of Pythagorean fuzzy set and β-subalgebras. One can further study on rough fuzzy, rough Pythagorean fuzzy and Tripolar Pythagorean fuzzy sub structures by connecting with on β-algebras.

REFERENCES

1. Atanasov KT. Intuitionistic fuzzy sets and systems. J Math Appl. 1986; 20(1):87–96.
2. Jenitha Vinnarasi R, Stanius and Arul Mary A, Pythagorean Fuzzy ideals in Subraction Algebras, Int. J. of Research Publication and Reviews, 2021; 2 (9): 477- 482.
3. Neggers J. and Kim Hee Sik, On β- algebras, Math.Solvaca, 2002, 52 (5), pp-517-530.
4. Sujatha K, Chandramouleeswaran M and Muralikrishna P. On intuitionistic fuzzy β-subalgebras of β-algebras, Global J Pure Appl Math. 2010; 9(1):559–66.
5. Yager R.R, Pythagorean Fuzzy subsets, Int. Proc. Joint IFSA world Congress, Annual meeting, Edmomton, Canada, 2013, 57-61.
6. Yager R.R and A.M. Abbu Sov, Pythagorean membership grades, Complex numbers and decision making, Int.J.Intell Syst 28 (2013), 436-452.
7. Zadeh L.A. Fuzzy sets. Inform and Control. 1965; 8(3):338–53.

GENERALIZED JORDAN RIGHT DERIVATION ASSOCIATED WITH RIGHT(JORDAN RIGHT) DERIVATION ON SEMIRINGS

S. Kavitha

Assistant Professor, PG and Research Department of Mathematics,
Saiva Bhanu Kshatriya College,
Aruppukottai - 626101,Tamil Nadu, India.
E-mail:kavithasakthivelapk@gmail.com

ABSTRACT: *In this paper, we introduce the notion of Generalized Jordan right derivation associated with right (Jordan right) derivation on semirings. We discuss some definitions and examples, and also we prove some elegant results.*

Mathematics Subject Classification*: 16Y60*

Keywords*: Semirings, Jordan right derivation, generalized Jordan right derivation.*

1. INTRODUCTION

The notion of derivations on semirings has been introduced [1] by Jonathan Golan. Motivated by this, Chandramouleeswaran and Thiruveni [2] studied the notion of derivations on semirings. A Classical result of Herstein [3] asserts that any Jordan derivation on a 2-torsion free prime ring is a derivation. A brief proof of Herstein's theorem can be found in [4]. Cusack [5] generalized Herstein's theorem to a 2-torsion free semiprime ring[6]. In 1990, Bresar and Vukman [7] have introduced the notion of left derivation in rings, and also they introduced the notion of generalized derivations on rings. Ashraf and Ali in [8] introduced the definitions of generalized left derivation (generalized Jordan left derivation) if there exists a Jordan left derivation on a ring. Motivated by this, Chandramouleeswaran and Nirmala Devi [9] discussed the notion of left derivation, generalized left derivation on semirings and also Chandramouleeswaran and Nirmala Devi [10] introduced the notion of right derivations on semirings. Motivated by this, in our work, we introduce the notion of Generalized Jordan right derivation associated with right (Jordan right) derivation on semirings and we prove some elegant results.

2. PRELIMINARIES

In this section, we recall some basic definitions and results that are required for our work.

Definition 2.1: A semiring is a nonempty set S on which two binary operations of addition + and multiplication $\cdot$ have been defined such that the following conditions are satisfied:

1. (S , +) is a monoid with identity element 0;
2. (S , $\cdot$) is a monoid with identity element 1_S;
3. Multiplication distributes over addition from either side:

   $a\cdot (b + c) = a\cdot b + a\cdot c$; $(b + c) \cdot a = b\cdot a + c\cdot a \ \forall a, b, c \in S$
4. $0\cdot s = 0 = s\cdot 0$ for all $s \in S$.

Definition 2.2: A Semiring (S, +, $\cdot$) is said to be additively commutative, if (S, +) is a commutative semigroup.

A Semiring (S, +, $\cdot$) is said to be multiplicatively commutative, if (S, $\cdot$) is a commutative semigroup.
The Semiring S is said to be a commutative semiring, if both additively and multiplicatively commutative.
Definition 2.3: A Semiring (S, +, $\cdot$) is additively cancellative, if it is both additively left and right cancellative.
A Semiring (S, +, $\cdot$) is a multiplicatively cancellative, if it is both multiplicatively left and right cancellative.
Definition 2.4: Let S be a semiring. A semiring S is said to be 2-torsion free, if 2a = 0, with a $\in S \Rightarrow$ a = 0.
Definition 2.5: Let S be a semiring. A right S- semimodule is a commutative monoid (X,+) with additive identity 0_x for which we have a function $X \times S \rightarrow X$, denoted by $(x, s) \rightarrow xs$ and called scalar multiplication, which satisfies the following conditions.
For all elements s and s' of S and all elements x and x' of X:
1. x (ss') = (xs)s'
2. (x +x') s = xs +x's
3. x (s +s') = xs +xs'
4. $x1_s = x$.

Definition 2.6: Let S be a semiring. An additive mapping d: $S \rightarrow S$ is called a derivation on S,
if $d(xy) = d(x)y + x\, d(y) \quad \forall\, x, y \in S$.
Let S be a semiring. An additive mapping d: $S \rightarrow S$ is called a Jordan derivation on S,
if $d(x^2) = d(x)x + x\, d(x) \quad \forall\, x \in S$.
Definition 2.7: Let S be a semiring. An additive mapping F: $S \rightarrow S$ is called a generalized derivation, if there exists a derivation d: $S \rightarrow S$ such that $F(xy) = F(x)y + x\, d(y) \quad \forall\, x, y \in S$.
Let S be a semiring. An additive mapping F: $S \rightarrow S$ is called a generalized Jordan derivation, if there exists a Jordan derivation d: $S \rightarrow S$ such that $F(x^2) = F(x)x + x\, d(x) \quad \forall\, x \in S$.

3. JORDAN RIGHT DERIVATION:

In this section, we discuss the notion of Jordan right derivation on semirings.
Definition 3.1: Let S be a semiring and X a S-module. An additive mapping d_R: $S \rightarrow X$ is called a Jordan right derivation on S, if $d_R(x^2) = 2\, d_R(x)x \quad \forall\, x \in S$.
Example 3.2:
Let S = $\left\{ \begin{pmatrix} a & b \\ 0 & a \end{pmatrix} : a,b \in Z^+ \right\}$ be a commutative semiring
and X = $\left\{ \begin{pmatrix} a & b \\ 0 & a \end{pmatrix} : a,b \in Z \right\}$ a S-module.
Define a map d_R: $S \rightarrow X$ given by $d_R\left(\begin{pmatrix} a & b \\ 0 & a \end{pmatrix} \right) = \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix}$
Then d_R is a Jordan right derivation on S.
Lemma 3.3: Let (S, +, $\cdot$) be an additively commutative semiring. Then the sum of two Jordan

right derivation on S is again a Jordan right derivation.

Proof:

Let S be an additively commutative semiring.

Let $d_{R1}, d_{R2} : S \to X$ be two Jordan right derivation.

Claim: $d_{R1} + d_{R2}$ is a Jordan right derivation on S.

$$(d_{R1} + d_{R2})(x^2) = (d_{R1})(x^2) + (d_{R2})(x^2)$$
$$= 2\, d_{R1}(x)\, x + 2\, d_{R2}(x)\, x \quad \forall x \in S.$$
$$= 2\, (d_{R1}(x) + d_{R2}(x))x \quad \forall x \in S.$$
$$(d_{R1} + d_{R2})(x^2) = 2\, (d_{R1} + d_{R2})(x)x \quad \forall x \in S.$$

$\therefore$ $d_{R1} + d_{R2}$ is a Jordan right derivation on S.

4. GENERALIZED JORDAN RIGHT DERIVATION:

In this section, we discuss the notion of generalized Jordan right derivation associated with right
(Jordan right) derivation on semirings and prove some elegant results.

Definition 4.1: Let S be a semiring and X a S-module. An additive mapping $F_R : S \to X$ is called a **generalized right derivation associated with right derivation**, if there exists a right derivation $d_R : S \to X$ such that $F_R(xy) = F_R(x)y + d_R(y)x \quad \forall x,y \in S$.

Example 4.2:

Let $S = \left\{ \begin{pmatrix} 0 & 0 & 0 \\ a & 0 & 0 \\ b & c & 0 \end{pmatrix} : a,b,c \in Z^+ \right\}$ be a semiring

and $X = \left\{ \begin{pmatrix} 0 & 0 & 0 \\ a & 0 & 0 \\ b & c & 0 \end{pmatrix} : a,b,c \in Z \right\}$ a S-module.

Define a map $F_R : S \to X$ such that $F_R\left(\begin{pmatrix} 0 & 0 & 0 \\ a & 0 & 0 \\ b & c & 0 \end{pmatrix} \right) = \begin{pmatrix} 0 & 0 & 0 \\ a & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \forall a, b, c \in Z^+$.

Then there exists a right derivation $d_R : S \to X$ such that

$$d_R\left(\begin{pmatrix} 0 & 0 & 0 \\ a & 0 & 0 \\ b & c & 0 \end{pmatrix} \right) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ b & 0 & 0 \end{pmatrix} \quad \forall a, b, c \in Z^+.$$

Then F_R is a generalized right derivation associated with right derivation on S.

Definition 4.3: Let S be a semiring and X a S-module. An additive mapping $F_R : S \to X$ is called a generalized right derivation associated with Jordan right derivation, if there exists a Jordan right derivation $d_R : S \to X$ such that $F_R(xy) = F_R(x)y + d_R(y)x \ \forall x, y \in S$.

Example 4.4:

Let S = $\left\{ \begin{pmatrix} 0 & a & b \\ 0 & 0 & a \\ 0 & 0 & 0 \end{pmatrix} : a, b \in Z^+ \right\}$ be a semiring

and X = $\left\{ \begin{pmatrix} 0 & a & b \\ 0 & 0 & a \\ 0 & 0 & 0 \end{pmatrix} : a, b \in Z \right\}$ a S-module.

Define a map $F_R : S \to X$ such that

$$F_R\left(\begin{pmatrix} 0 & a & b \\ 0 & 0 & a \\ 0 & 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & 0 & b \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \forall\, a, b \in Z^+.$$

Then there exists a Jordan right derivation $d_R : S \to X$ such that

$$d_R\left(\begin{pmatrix} 0 & a & b \\ 0 & 0 & a \\ 0 & 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & a & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \forall\, a, b \in Z^+.$$

Then F_R is a generalized right derivation associated with Jordan right derivation on S.

Definition 4.5: Let S be a semiring and X a S-module. An additive mapping $F_R : S \to X$ is called a generalized Jordan right derivation associated with right derivation, if there exists a right derivation $d_R : S \to X$ such that $F_R(x^2) = F_R(x)x + d_R(x)x \ \ \forall\, x \in S$.

Definition 4.6: Let S be a semiring and X a S-module. An additive mapping $F_R : S \to X$ is called a generalized Jordan right derivation associated with Jordan right derivation, if there exists a Jordan right derivation $d_R : S \to X$ such that $F_R(x^2) = F_R(x)x + d_R(x)x \ \ \forall\, x \in S$.

Example 4.7: The mappings $\mathbf{F_R}$ and $\mathbf{d_R}$ given in example 4.4, are generalized Jordan right derivation associated with right (Jordan right) derivation on the given semiring.

Remark 4.8: Every generalized right derivation associated with Jordan right derivation on a Semiring S is a generalized Jordan right derivation associated with Jordan right derivation but the converse need not be true.

Lemma 4.9: Let $(S, +, \cdot)$ be an additively commutative semiring. Sum of two generalized right derivation associated with Jordan right derivation on S is again a generalized right derivation associated with Jordan right derivation.

Proof: Let S be an additively commutative semiring. Let $F_{R1}, F_{R2} : S \to X$ be a generalized right derivation associated with Jordan right derivation.

To prove: $F_{R1} + F_{R2}$ is a generalized right derivation associated with Jordan right derivation

$$\begin{aligned} F_{R1}(xy) + F_{R2}(xy) &= F_{R1}(x)y + d_{R1}(y)x + F_{R2}(x)y + d_{R2}(y)x \\ &= (F_{R1} + F_{R2})(x)y + (d_{R1} + d_{R2})(y)x \end{aligned}$$

$F_{R1}(xy) + F_{R2}(xy) = (F_{R1} + F_{R2})(x)y + (d_{R1} + d_{R2})(y)x$

$\therefore$ $F_{R1} + F_{R2}$ is a generalized right derivation associated with Jordan right derivation.

Lemma 4.10: Let $(S, +, \cdot)$ be an additively commutative semiring. Sum of two generalized Jordan right derivation associated with Jordan right derivation on S is again a generalized Jordan right derivation associated with Jordan right derivation.

Proof: Let S be an additively commutative semiring.

Let $F_{R1}, F_{R2} : S \to X$ be a generalized Jordan right derivation associated with Jordan right derivation.

To prove: $F_{R1} + F_{R2}$ is a generalized Jordan right derivation associated with Jordan right derivation.

$$(F_{R1} + F_{R2})(x^2) = F_{R1}(x^2) + F_{R2}(x^2)$$

$$= F_{R1}(x)x + d_{R1}(x)x + F_{R2}(x)x + d_{R2}(x)x$$

$$= (F_{R1} + F_{R2})(x)x + (d_{R1} + d_{R2})(x)x$$

$\therefore$ $F_{R1} + F_{R2}$ is a generalized Jordan right derivation associated with Jordan right derivation.

Lemma 4.11: Let S be a multiplicatively cancellative semiring. Suppose that $a \in S$ such that $(ax)a = (xa)a$ $\forall x \in S$, then $a \in Z(S)$.

Proof: Let S be a multiplicatively cancellative semiring.

Claim: $a \in Z(S)$

Suppose $(ax)a = (xa)a$ $\forall x \in S$……….. (1) Replace x by xr in (1) , we get

$(a(xr))a = ((xr)a)a$ $\forall x \in S$

$(ax)ra = (xa)ra$ ………. (2)

Since S is a multiplicatively cancellative semiring, $ax = xa$, $\forall x \in S$

$\therefore$ $a \in Z(S)$

Theorem 4.12: Let S be a 2-torsion free prime semiring. Let X be a S-module such that $aSx = 0 \Rightarrow a = 0$ or $x = 0$ $\forall a \in S, x \in X$. If S admits a generalized right derivation F_R associated with a non zero Jordan right derivation d_R , then S is commutative.

REFERENCES:

[1] Jonathan S.Golan, Semirings and their Applications, Kluwer Acadamic Press (1969).

[2] Chandramouleeswaran. M and Thiruveni. V, On derivations of semirings, Advances in Algebras Vol (3) (2010),123-131.

[3] Herstein.I.N, Jordan derivations of prime rings, Proc. Amer. Math. Soc.8 (1957), 1104-1119.

[4] Bresar.M and Vukman.J, Jordan derivations on prime rings, Bull. Austral. Math. Soc.37 (1988), 321-322.

[5] Cusack.J, Jordan derivations on semiprime rings, Proc. Amer. Math. Soc.53 (1975), 321-324.

[6] Bresar.M, Jordan derivations on semiprime rings, Proc. Amer. Math. Soc.104 (1988), 1003-1006.

[7] Bresar.M and Vukman.J, On left derivations and related mappings , Proc. Amer. Math. Soc.Vol 110, No:1 Sep (1990), 7-16.
[8] Ashraf.M and Ali.S: On generalized Jordan left derivations in rings, Bull.Korean Math. Soc.45 (2) (2008), 253-261.
[9] Chandramouleeswaran.M and Nirmala Devi. S.P: Generalized left derivation on semirings, IJMA-4 (10), Oct-2013, 159-164.
[10] Chandramouleeswaran.M and Nirmala Devi. S.P: Right derivations on Semirings, International Mathematical Forum Vol.8, 2013, No.32, 1569-1576.

CONNECTED BOUNDARY WEIGHT DOMINATION ON S-VALUED GRAPHS

A. Arul Devi

Sri Ramanas College Of Arts And Science For Women
Chidhambarapuram, Tamilnadu. India.
E-Mail: aruldevika.22@gmail.com.

Abstract: *In the year 2015, Chandramouleeswaran and others introduced the notion of semiring valued graphs (briefly called S-valued graphs). In the same year, Jeyalakshmi, in her work, discussed the concept of vertex domination of S-valued graphs. K.M. Kathiresan, G.Marimuthu and M. Sivanantha Saraswathi, introduced the boundary domination in graphs. Mohammed Alatif, Putaswamy and Nayaka introduced the concept of connected boundary domination in graphs Motivated by this, in this paper, we discuss the Connected Boundary Weight Domination On S-Valued Graphs.*

Keywords: *S – Valued graphs, Weight domination, Boundary neighbourhood, Boundary degree, Boundary domination number , Connected boundary weight domination number.*
AMS Classification: *16Y60,05C25,05C76*

1. INTRODUCTION :

P Sampathkumar and Walikar introduced the concept of connected domination in graphs[14]. Kathiresan and others introduced the concept of boundary domination in graphs[9]. Putaswamy and Mohammed introduced the concept of boundary edge domination in graphs[13]. Mohammed Alatif, Putaswamy and Nayaka introduced the concept of connected boundary domination in graphs[12]. Chandramouleeswaran and others introduced the concept of S-valued graphs[13]. Jeyalakshmi and Chandramouleeswaran introduced the concept of vertex domination in S-valued graphs[4]. Mangalalavanya and Chandramouleeswaran introduced the concept of edge domination in S-valued graphs[10]. Arul Devi and Thiruveni introduced the concept of Boundary weight domination on S-valued graphs[1]. In this paper we introduce the concept of Connected boundary weight domination on S-valued graphs.

2. PRELIMINARIES :

Definition 2.1. [9]

Let G be a simple graph G=(V, E) with vertex set $V(G) = \{v_1, v_2, \dots, v_n\}$. For $i \neq j$, a vertex v_i is a boundary vertex of v_j if $d(v_i, v_t) \leq d(v_j, v_t)$forall $v_t \in N(v_i)$. A vertex v is called a boundary neighbour of u if v is a nearest boundary of u. If $u \in V$, then the boundary neighbourhood of u denoted by $N_b(u)$ is defined as $N_b(u) = \{v \in V : d(u, w) \leq d(u, u)\ forall\ w \in N(u)\}$

Definition 2.2 [6]

A subset S of V(G) is called a boundary dominating set if every vertex of V-S is boundary dominated by some vertex of S. The minimum taken over all boundary dominating sets of a graph G is called the boundary domination number on G and is denoted by $\gamma_b(G)$.

Definition 2.3.[8]

A boundary dominating set S of a connected graph G is called the connected boundary dominating set if the induced subgraph <S> of G is connected. The minimum cardinality of a cb-set is called the connected boundary domination number $\gamma_{cb}(G)$.

Definition 2.4 [6]

Let $G^S = (V, E, \sigma, \psi)$ be a S-valued graph by V_S mean the set $V\ X\ S$ and E_S mean the set $E\ X\ S$ any element of V_S will be denoted by $v_i(s_i)$ where $v_i \in V$ and $s_i \in S$ forall $i = 1,2,\dots,n$. Similarly, any element of E_S will be denoted by $e_i^j(s_{i,j})$ where $e_i^j = (v_i, v_j) \in E\ and\ s_{ij} = \min\{s_i,\ s_j\}$.

Definition 2.5 [4]

Consider the S-valued graph $G^S = (V_S, E_S)$. where $V_S = \{\ v_i(s_i)/v_i \in V\ and\ s_i \in S\} and\ E_S = \{e_i^j(s_{i,j})\}$

- The order of G^S is defined as $p_S = (|V|_{S,}\ |V|)$
- The size of G^S is defined as $q_S = (|E|_{S,}\ |E|)$
- The open neighbourhood of v_i in G^S is defined as $N_S(v_i) = \{(v_j, \sigma(v_j))|(v_i, v_j) \in E, \psi(v_i, v_j) \in S\}$.
- The closed neighbourhood of v_i in G^S is defined as $N_S[v_i] = N_S(v_i) \cup \{v_i, \sigma(v_i))\}$.

Analogously, we can define the open(closed) neighbourhood of an edge in G^S.

Definition 2.6:[1]

Consider the S-valued graph $G^S = (V_S, E_S)$ where $V_S = \{v_1(s_1),\ v_2(s_2), \dots\dots, v_n(s_n)\}$. For $i \neq j$, a vertex $v_i(s_i)$ is said to be a boundary vertex of $v_j(s_j)$ if $dist_S(v_i(s_i)\ , v_t(s_t)) \leq dist_S(v_j(s_j)\ , v_t(s_t))$ forall $v_t(s_t) \in N_S(v_j(s_j))$.

Definition 2.7:[1]

Consider the S-valued graph $G^S = (V_S, E_S)$ where $V_S = \{v_1(s_1),\ v_2(s_2), \dots\dots, v_n(s_n)\}$. A vertex $v_i(s_i) \in V_S$ is called boundary neighbour of a vertex $v_j(s_j) \in V_S$ if $v_i(s_i)$ is a nearest boundary of $v_j(s_j)$. If $v_j(s_j) \in V_S$, then the boundary neighbourhood of $v_j(s_j)$, denoted by $bN_S(v_j(s_j)$, is defined to be the set $bN_S(v_j(s_j)) = \{v_i(s_i) \in V_S/dist_S(v_i(s_i)\ , v_t(s_t)) \leq\ dist_S(v_j(s_j)\ , v_t(s_t))$ forall $v_t(s_t) \in N_S(v_j(s_j))\}$.

Definition 2.8:[1]

The boundary degree of a vertex $v_j(s_j) \in V_S$, denoted by $bdeg_S(v_j(s_j) = (|bN_S(v_j(s_j))|_S, |bN(S)|)$. The maximum and minimum boundary degree of the graph G^S are denoted respectively be,

$$\Delta_S^b(G^S) = max_{v_j(s_j)\in V_S}(|bN_S(v_j(s_j))|_S, |bN_S(v_j(s_j))|)$$

$$\delta_S^b(G^S) = min_{v_j(s_j)\in V_S}(|bN_S(v_j(s_j))|_S, |bN_S(v_j(s_j))|)$$

Definition 2.9:[1]A vertex $v_j(s_j)$ is said said to be a boundary weight dominating vertex of a vertex $v_i(s_i)$ if $v_i(s_i)$ is a boundary neighbour of $v_j(s_j)$. A subset $D_S \subseteq V_S$ is called boundary weight dominating set if every vertex $V_S - D_S$ is boundary weight dominated by some vertex of D_S. The minimum cardinality of all boundary weight dominating set of a graph G^S is called boundary weight domination number of G^S and is denoted by $\gamma_b^S(G^S)$.

3. CONNECTED BOUNDARY WEIGHT DOMINATION ON S-VALUED GRAPHS

In this section we introduce the notion of connected boundary weight domination in S-valued graphs.

Definition 3.1:

A boundary weight dominating set D^S of a connected S-valued graph G^S is called the connected boundary weight dominating set, if the induced subgraph <H^S> of G^S is connected. The minimum cardinality of a connected boundary weight dominating set is called the connected boundary weight domination number, and it is denoted by $\gamma_{cb}^S(G^S)$.

Example 3.2:

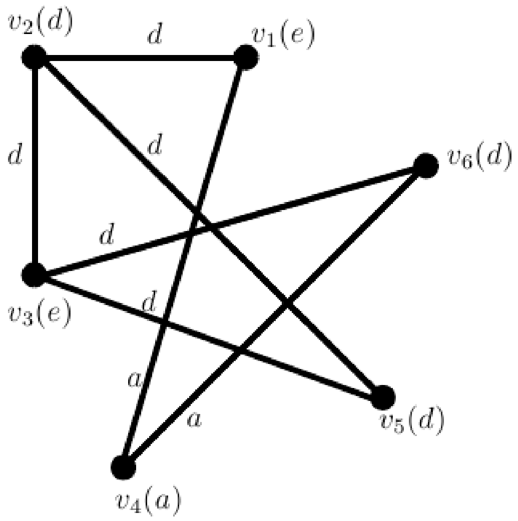
Consider the semiring S=({0,a,b,c,d,e},+,·,≼) with the following Cayley tables.

+	0	a	b	c	d	e
0	0	a	b	c	d	e
a	a	a	b	c	d	e
b	b	b	b	d	d	e
c	c	c	d	c	d	e
d	d	d	d	d	d	e
e	e	e	e	e	e	e

·	0	a	b	c	d	e
0	0	0	0	0	0	0
a	0	0	0	a	a	e
b	0	a	b	a	b	b
c	0	0	0	c	c	e
d	0	a	b	c	d	e
e	0	c	e	c	e	e

⪯	Elements of S
0	0, a, b, c, d, e
a	a, b, c, d, e
b	b, d, e
c	c, d, e
d	d, e
e	e

Consider the S-valued graph $G^S = (V_S, E_S)$,



One can easily verify that the boundary degree of each vertices of G^S as follows:

$bdeg_S(v_1(e)) = (e, 2), bdeg_S(v_2(d)) = (d, 2), bdeg_S(v_3(e)) = (e, 2),$

$$bdeg_S(v_4(a)) = (e, 3), bdeg_S(v_5(d)) = (e, 3), bdeg_S(v_6(d)) = (d, 2)$$

$H^S = \{v_3(e), v_2(d), v_6(d)\}$ is a connected boundary weight dominating vertex set.

The minimum cardinality of a connected boundary weight dominating set of $\gamma_{cb}^S(G^S) = (e, 3)$

The maximum degree of G^S is $\Delta_S^b(G^S) = (e, 3)$ and minimum degree of G^S is $\gamma_{cb}^S(G^S) = (d, 2)$

Theorem 3.6:

For any path $P_n^S, n \geq 3$,the boundary weight dominating vertex number is $\gamma_b^S(P_n^S) = (|D|_S, n-2)$.

Proof: Consider the path P_3^S is given by, $v_1(s_1), e_1^2(s_{12}), v_2(s_2)e_2^3(s_{23}), v_3(s_3)$.

$$P_{v_1v_2}^S = \{v_1(s_1), e_1^2(s_{12}), v_2(s_2)\}.\ w\left(P_{v_1v_2}^S\right) = \sum \psi(e_1^2) = \psi(e_1^2)\ and\ l\left(P_{v_1v_2}^S\right) = 1.$$

$$P_{v_1v_3}^S = \{v_1(s_1), e_1^2(s_{12}), v_2(s_2)e_2^3(s_{23})v_3(s_3)\}.\ w\left(P_{v_1v_3}^S\right) = \sum \psi(e_1^2) + \psi(e_2^3)\ and\ l\left(P_{v_1v_2}^S\right) = 2.$$

$$dist_S\left(v_1(s_1), v_2(s_2)\right) = \min\{w\left(P_{v_1v_2}^S\right), l\left(P_{v_1v_2}^S\right)\} = (\psi(e_1^2), 1);$$

$$dist_S\left(v_1(s_1), v_3(s_3)\right) = \min\{w\left(P_{v_1v_3}^S\right), l\left(P_{v_1v_3}^S\right)\} = (\psi(e_1^2) + \psi(e_2^3), 2);$$

$$P_{v_2v_3}^S = \{v_2(s_2)e_2^3(s_{23}), v_3(s_3)\}.\ w\left(P_{v_2v_3}^S\right) = \sum \psi(e_2^3) = \psi(e_2^3)\ and\ l\left(P_{v_2v_2}^S\right) = 1.$$

$$dist_S\left(v_2(s_2), v_3(s_3)\right) = \min\{w\left(P_{v_2v_3}^S\right), l\left(P_{v_2v_3}^S\right)\} = (\psi(e_3^3), 1)$$

Let $D^S = \{v_2(s_2)\}. Then\ V_S - D_S = \{v_1(s_1), v_2(s_2), v_3(s_3)\}$ is boundary weight dominated by D^S. If every vertex D^S is a connected boundary weight dominating vertex set. Therefore $\gamma_{cb}^S(P_n^S) = (|D^S|_{S,}|D^S|) = (\sigma(v_2), 1) = (|D^S|_{S,} 1)$. where $n = 3$.

We know that, a vertex $v_i(s_i)$ will dominate $v_{i-1}(s_{i-1})$ and $v_{i+1}(s_{i+1})$ in any path P_n^S, where $n = 3$. Proceeding like this, the connected boundary weight domination number of the S-valued path $\gamma_b^S(P_4^S) = (|D^S|_S, 2)$. In general, the connected boundary weight domination number of the S-valued path $\gamma_b^S(P_n^S) = (|D^S|_S, n-2)$. where $n \geq 5$, where $D^S$is a connected boundary weight dominating vertex set.

Theorem. 3.7:

For any complete S-valued graph , $K_n^S, n \geq 4$, the connected boundary weight dominating vertex number is

$$\gamma_b^S(K_n^S) = (|D_S|_S, 1).$$

Proof: Consider the complete graph K_n^S, is given by the path,

$$v_1(s_1), e_1^2(s_{12}), v_2(s_2)e_2^3(s_{23}), v_3(s_3)e_3^4(s_{34})v_4(s_4)e_4^2(s_{42})v_2(s_2)e_1^2(s_{12})v_1(s_1)e_1^3(s_{13})v_3(s_3)$$

$$P_{v_1v_2}^S = \{v_1(s_1)\ e_1^2(s_{12})\ v_2(s_2), v_1(s_1)\ e_1^3(s_{13})v_3(s_3)e_2^3(s_{23})\ e_3^4(s_{34})v_2(s_2), v_1(s_1)e_1^4(s_{14})\ v_4(s_4)e_3^4(s_{34})v_3(s_3)e_2^3(s_{23})v_4(s_4)v_2(s_2)\}$$

$$w\left(P_{v_1v_2}^S\right) = \{\psi(e_1^2) + \psi(e_1^3) + \psi(e_1^3) + \psi(e_2^3) + +\psi(e_3^4)\}and\ l\left(P_{v_1v_3}^S\right) = \min(1, 2, 3) = 1$$

$$P_{v_1v_3}^S = \{v_1(s_1)\ e_1^3(s_{13})\ v_3(s_3), v_1(s_1)e_1^4(s_{14})v_4(s_4)e_3^4(s_{34})v_3(s_3), v_1(s_1)e_1^2(s_{12})\ v_2(s_2)e_2^3(s_{23})v_3(s_3)\}$$

$$w\left(P_{v_1v_3}^S\right) = \{\psi(e_1^3) + \psi(e_1^4) + \psi(e_4^3) + \psi(e_1^2) + \psi(e_2^3)\}and\ l\left(P_{v_1v_3}^S\right) = \min(1, 2, 3) = 1$$

$$P_{v_1v_4}^S = \{v_1(s_1)\ e_1^4(s_{14})\ v_4(s_4), v_1(s_1)e_1^3(s_{13})v_3(s_3)e_3^4(s_{34})v_4(s_4), v_1(s_1)e_1^2(s_{12})\ v_2(s_2)e_2^3(s_{23})v_3(s_3)e_3^4(s_{34})v_4(s_4)\}$$

$$w\left(P_{v_1v_3}^S\right) = \{\psi(e_1^4) + \psi(e_1^3) + \psi(e_3^4) + \psi(e_1^2) + \psi(e_2^3)\}and\ l\left(P_{v_1v_4}^S\right) = \min(1, 2, 3) = 1$$

$$dist_S(v_1(s_1), v_2(s_2)) = \min(\psi(e_1^2) + \psi(e_1^3) + \psi(e_1^3) + \psi(e_2^3) + +\psi(e_3^4), 1);$$

$$dist_S(v_1(s_1), v_3(s_3)) = \min(\psi(e_1^3) + \psi(e_1^4) + \psi(e_4^3) + \psi(e_1^2) + \psi(e_2^3), 1);$$

$$dist_S(v_1(s_1), v_4(s_4)) = \min(\psi(e_1^4) + \psi(e_1^3) + \psi(e_3^4) + \psi(e_1^2) + \psi(e_2^3), 1).$$

$$bN_S(v_1(s_1) = \{v_2(s_2), v_3(s_3), v_4(s_4)\}; bdegN_S(v_1(s_1) = \{s_2 + s_3 + s_4, 3\}$$

Let $D_{1S} = \{v_1(s_1)\}, V_S - D_{1S} = \{v_2(s_2), v_3(s_3), v_4(s_4)\}. Every$ vertex in $V_S - D_{1S}$ $is\ dominated$ by D_{1S}, $D_{1S}$is a connected boundary weight dominating vertex set provided $\sigma(s_1) = s_1, s_2, s_3, s_4 \preccurlyeq s_1, \gamma_b^S(D_{1S}) = (|D_{1S}|_S, 1)$.

Proceeding like this, we can find the other vertices of the bounded neighbourhood is,

$bN_S(v_2(s_2) = \{v_1(s_1), v_3(s_3), v_4(s_4)\}; bdegN_S(v_2(s_2) = \{s_1 + s_3 + s_4, 3\}$

Let $D_{2S} = \{v_2(s_2)\}, V_S - D_{2S} = \{v_1(s_1), v_3(s_3), v_4(s_4)\}. Every$ vertex in $V_S - D_{2S}$ $is\ dominated$ by $D_{2S,}$ $D_{2S,}$ is a connected boundary weight dominating vertex set provided $\sigma(s_2) = s_1, s_2, s_3, s_4 \preccurlyeq s_2, \gamma_b^S(D_{2S}) = (|D_{2S}|_S, 1)$.

$bN_S(v_3(s_3) = \{v_2(s_2), v_1(s_1), v_4(s_4)\}; bdegN_S(v_3(s_3) = \{s_1 + s_2 + s_4, 3\}$

Let $D_{3S} = \{v_3(s_3)\}, V_S - D_{3S} = \{v_1(s_1), v_2(s_2), v_4(s_4)\}. Every$ vertex in $V_S - D_{3S}$ $is\ dominated$ by D_{3S}, D_{3S},is a connected boundary weight dominating vertex set provided $\sigma(s_3) = s_1, s_2, s_3, s_4 \preccurlyeq s_3, \gamma_b^S(D_{3S}) = (|D_{3S}|_S, 1)$.

$bN_S(v_4(s_4) = \{v_2(s_2), v_1(s_1), v_3(s_3)\}; bdegN_S(v_4(s_4) = \{s_1 + s_2 + s_3, 3\}$

Let $D_{4S} = \{v_4(s_4)\}, V_S - D_{4S} = \{v_1(s_1), v_2(s_2), v_3(s_3)\}. Every$ vertex in $V_S - D_S$ $is\ dominated$ by D_{4S} , D_{4S} is a connected boundary weight dominating vertex set provided $\sigma(s_4) = s_1, s_2, s_3, s_4 \preccurlyeq s_4, \gamma_b^S(D_{4S}) = (|D_{4S}|_S, 1)$.

Thus we conclude that for K_4^S, then the set $D_S = \{v_i(s_i)\}$ will be a connected boundary weight dominating vertex set if $s_j \preccurlyeq s_i$, forall $i \neq j$.

Since any vertex in a complete graph K_n^S will be dominated all vertices preceeding as above we conclude that $\gamma_{cb}^S(K_n^S$)= $(|D^S|_S, 1)$ for some $s_i \in S$ such that $s_j \preccurlyeq s_i$, forall $i \neq j$

Theorem 3.8:

For any complete bipartite S-valued graph, then the connected boundary weight dominating vertex number $\gamma_{cb}^S(K_{mn}^S$)= $(|D^S|_S, 2)$.

Proof:

Let V_{1S} and V_{2S} be partition of the vertex set of K_{mn}^S. Let Then $v_1(s_1) \in V_{1S}. Then$ $dist_S(v_1(s_1), v_2(s_2)) = (\psi(e_1^2) + \psi(e_2^3), 2)$ forall $v_2(s_2) \in V_{1S} - \{v_1(s_1)\}$ and every vertex $v_2(s_2)\ in\ V_{1S}$ is a boundary vertex $v_1(s_1)$ of except $v_1(s_1)$. Similarly if $u_1(s_1) \in V_{2S}$, then every vertex of $v_2(s_2)$ is a connected boundary neighbour of $u_1(s_1)$ except $u_1(s_1)$. Thus $\gamma_{cb}^S(K_{mn}^S$)= $(|D_S|_S, 2$.

Theorem 3.9:

Let T^S be a S-valued tree of order $(|V|_S, n)$ with $(|V_1|_S, n_1)$ pendent vertices. Then the connected boundary weight dominating vertex number is $\gamma_{cb}^S(T^S$)= $(|D^S|_S, n - n_1)$.

Proof: Let V_{1S} be the set of all pendent vertices of the tree T^S of order $(|V_1|_S, n_1)$. Then every vertex in $V_S - V_{1S}$ has a maximum weight and boundary neighbor in V_{1S}.Then the connected boundary weight dominating vertex number is $\gamma_{cb}^S(T^S$)= $(|D^S|_S, n - n_1)$.

Theorem 3.10:

For any connected S-valued graph G^S, $\gamma_b^S(G^S) \leq \gamma_{cb}^S(G^S)$.

Proof:

Every connected boundary weight dominating set of a graph G^S is also a boundary weight dominating vertex set. The boundary weight dominating vertex set is need not be connected. So the minimum number of vertices are dominated by the graph G^S. But the connected boundary weight dominating vertex set must be connected.

Hence $\gamma_b^S(G^S) \leq \gamma_{cb}^S(G^S)$.

4. CONCLUSION:

In this paper, we have studied the notion of connected boundary weight dominating vertex sets and connected boundary weight dominating vertex number of S-valued graphs. Further, we have introduced the notion of connected boundary weight dominating polynomials of a given S-valued graphs and determined the same for certain class of S-valued graphs. In future, we have proposed to study the boundary edge weight domination on S-valued graphs and boundary edge weight domination number on S-valued graphs.

REFERENCES:

[1] Arul Devi A and Thiruveni V, Boundary weight domination on S-valued graphs, Indian Journal of Natural Sciences, (IJONS) vol-12, Issue-70, Feb 2022.

[2] Godsil C and Royle G, Algebraic Graph Theory , Springer-Verlag, New York, 2001.

[3] Jeyalakshmi. S. and Chandramouleeswaran. M Connected S-Valued graphs, Mathematical Sciences International Research Journal, ISSN : 2278-8697. Vol 4, Issue 2,2015,pp. 323-325.

[4] Jeyalakshmi. S. and Chandramouleeswaran. M Vertex Domination on S-Valued graphs, IOSR Journal of Mathematics., Vol 12, 2016, PP 08-12.

[5] Jeyalakshmi. S. and Chandramouleeswaran. M Diameter on S-Valued graphs, Mathematical Sciences International Research Journal, 2017, 6, 121-123.

[6] Jeyalakshmi. S. and Chandramouleeswaran. M Degree regular on S-Valued graphs, Mathematical Sciences Engineering Applications 2015, 4, 326-328.

[7] Jeyalakshmi. S., Rajkumar. M and Chandramouleeswaran. M Regularity of S-Valued graphs, Global Journal of Pure Applied Mathematics., Vol 2, 2015, PP 2971-2978.

[8] Jonathan Golan Semiring and their Applications, Kluwer Academic Publishers, London.

[9] Kathiresan K.M, Marimuthu G and Sivanantha Saraswathi M, Boundary Domination in Graphs, Kragujevac j. Math. 33(2010) 63-70.

ON INTUITIONISTIC FUZZY H-IDEALS IN Z-ALGEBRAS

S. Sowmiya* and M. Chandramouleeswaran**

* Assistant Professor, Department of Mathematics,
Sri Ramakrishna Engineering College, Coimabatore-22.
E-mail: vinayagarphd@gmail.com
** M.Chandramouleeswaran, Associate Professor and Head (Retd.),
PG and Research Department of Mathematics,
Saiva Bhanu Kshatriya College, Aruppukottai 626 101.
E-mail: moulee59@gmail.com

ABSTRACT: *In this article, the concept of intuitionistic fuzzy H-Ideals in Z-algebras is presented and some of their features are studied. The Z-homomorphic image and inverse image of intuitionistic fuzzy H-ideals in Z-algebras are examined. In addition, the Cartesian product of intuitionistic fuzzy H-ideals in Z-algebras is also investigated.*
Keywords: *Z-algebra, H-ideal, intuitionistic fuzzy H-ideal*

1. INTRODUCTION:

Imai and Iseki [4,5] introduced two new classes of abstract algebras: BCK-algebras and BCI-algebras. In 2017, Chandramouleeswaran et al.[3] introduced the concept of Z-algebras as a new structure of algebra based on propositional calculus. The Z-algebra is not a generalization of BCK/BCI-algebras. In 1965, Zadeh [8] introduced the fundamental concept of a fuzzy set which is a generalization of an ordinary set. In 1986, the idea of "intuitionistic fuzzy set" was first published by Atanassov [1], as a generalization of the notion of fuzzy set. In addition to the membership function, the idea of an intuitionistic fuzzy set also includes a non-membership function. Since then, other researchers have examined intuitionistic fuzzy structures in various algebras. In our earlier paper [7], we introduced fuzzy H-ideals in Z-algebras. In this article, we define the notion of intuitionistic fuzzy H-ideals in Z-algebras and investigated some of their properties.

2. PRELIMINARIES:

In this section we recall some basic definitions that are needed for our work.

Definition 2.1[3]: A Z-algebra $(X,*,0)$ is a nonempty set X with a constant 0 and a binary operation $*$ satisfying the following conditions:

(Z1) $x*0=0$

(Z2) $0*x=x$

(Z3) $x*x=x$

(Z4) $x*y=y*x$, when $x \neq 0$ and $y \neq 0$ $\forall$ $x, y \in X$.

Definition 2.2[3]: Let $(X,*,0)$ be a Z-algebra and I be a subset of X. Then, I is called an **Z-ideal** of X, if it satisfies the following conditions: For all x, y in X,

i) $0 \in I$

ii) $x * y \in I$ and $y \in I \Rightarrow x \in I$

Definition 2.3[7]: Let $(X,*,0)$ be a Z-algebra and I be a subset of X. Then, I is called an **H-ideal** of X, if it satisfies the following conditions: For all x, y, z in X,

i) $0 \in I$

ii) $x*(y*z) \in I$ and $y \in I \implies x*z \in I$

Definition 2.4[3]: Let $(X,*,0)$ and $(Y,*',0')$ be two Z-algebras. A mapping $h:(X,*,0) \to (Y,*',0')$ is said to be a **Z-homomorphism** of Z-algebras if $h(x*y) = h(x) *' h(y)$ for all $x, y \in X$.

Definition 2.5[3]: Let $h:(X,*,0) \to (Y,*',0')$ be a Z-homomorphism of Z-algebras. Then

1. h is called a **Z-monomorphism** of Z-algebras if h is 1-1.
2. h is called an **Z-epimorphism** of Z-algebras if h is onto.
3. h is called an **Z-endomorphism** of Z-algebras if h is a mapping from $(X,*,0)$ into itself.

Note : If $h:(X,*,0) \to (Y,*',0')$ is a Z-homomorphism then $h(0) = 0'$.

Definition 2.6[1]: Let X be a nonempty universal set. A fuzzy set A in X is characterized by a membership function μ_A which associates with each point x in X, a real number $\mu_A(x)$ in the interval [0,1] with $\mu_A(x)$ representing the "grade of membership" of x in A.

That is, a fuzzy set A in X is characterized by a membership function $\mu_A : X \to [0,1]$.

Definition 2.7[7]: Let $(X,*,0)$ be a Z-algebra. A fuzzy set A in X with membership function μ_A is said to be **fuzzy H - ideal** of a Z-algebra X if it satisfies the following conditions: For all x, y, z in X,

(i) $\mu_A(0) \geq \mu_A(x)$

(ii) $\mu_A(x*z) \geq \min\{\mu_A(x*(y*z)), \mu_A(y)\}$

Definition 2.8[1]: An **Intuitionistic Fuzzy Set** (IFS) A in a nonempty set X is an object having the form $A = \{\langle x, \mu_A(x), \beta_A(x)\rangle \mid x \in X\}$ where $\mu_A : X \to [0,1]$ denote the degree of membership and $\beta_A : X \to [0,1]$ denote the degree of non-membership functions such that for each $x \in X$ to the set A with $0 \leq \mu_A(x) + \beta_A(x) \leq 1$. For the sake of simplicity, we shall use the symbol $A = (\mu_A, \beta_A)$ for the IFS $A = \{\langle x, \mu_A(x), \beta_A(x)\rangle \mid x \in X\}$.

Definition 2.9[1]: If $A = \{\langle x, \mu_A(x), \beta_A(x)\rangle | x \in X\}$ and $B = \{\langle x, \mu_B(x), \beta_B(x)\rangle | x \in X\}$ be any two intuitionistic fuzzy set of a nonempty set X. Then,

1. $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$ and $\beta_A(x) \geq \beta_B(x)$ for all $x \in X$
2. $A = B$ iff $\mu_A(x) = \mu_B(x)$ and $\beta_A(x) = \beta_B(x)$ for all $x \in X$
3. $A^c = \{\langle x, \beta_A(x), \mu_A(x)\rangle | x \in X\}$
4. $A \cap B = \{\langle x, \mu_{A\cap B}(x), \beta_{A\cup B}(x)\rangle \mid x \in X\} = \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\beta_A(x), \beta_B(x))\rangle | x \in X\}$
5. $A \cup B = \{\langle x, \mu_{A\cup B}(x), \beta_{A\cap B}(x)\rangle \mid x \in X\} = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\beta_A(x), \beta_B(x))\rangle | x \in X\}$

6. $\oplus A = (\mu_A, (\mu_A)^c) = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle | x \in X\}$
7. $\otimes A = ((\beta_A)^c, \beta_A) = \{\langle x, 1 - \beta_A(x), \beta_A(x)\rangle | x \in X\}$

8. $\bigcap_{i\in\Omega} A_i = \left\{\left\langle x, \mu_{\bigcap_{i\in\Omega} A_i}(x), \beta_{\bigcup_{i\in\Omega} A_i}(x)\right\rangle \middle| x \in X\right\}$

where $\mu_{\bigcap_{i\in\Omega} A_i}(x) = \inf_{i\in\Omega}(\mu_{A_i}(x))$ and $\beta_{\bigcup_{i\in\Omega} A_i}(x) = \sup_{i\in\Omega}(\beta_{A_i}(x))$.

Definition 2.10[2]: Let $A = (\mu_A, \beta_A)$ be an intuitionistic fuzzy set in a nonempty set X. For $s, t \in [0,1]$, $U(\mu_A; s) = \{x \in X \mid \mu_A(x) \geq s\}$ is called an **upper s-level subset of A** and $L(\beta_A; t) = \{x \in X \mid \beta_A(x) \leq t\}$ is called the **lower t-level subset of A**.

Definition 2.11[2]: An IFS A in a set X with the degree of membership $\mu_A : X \to [0,1]$ and the degree of non-membership $\beta_A : X \to [0,1]$ is said to have **sup-inf property** if for any subset T of X there exists $x_0 \in T$ such that $\mu_A(x_0) = \sup_{t\in T} \mu_A(t)$ and $\beta_A(x_0) = \inf_{t\in T} \beta_A(t)$.

Definition 2.12[6]: Let h be any function from a set X into a set Y.

(i) Let $A = \{\langle x, \mu_A(x), \beta_A(x)\rangle \mid x \in X\}$ be an intuitionistic fuzzy set in X. Then image of A under h, denoted by $h(A) = \{\langle y, \mu_{h(A)}(y), \beta_{h(A)}(y)\rangle \mid y \in Y\}$ is an intuitionistic fuzzy set in Y, defined by:

$$\mu_{h(A)}(y) = \begin{cases} \sup\limits_{z\in h^{-1}(y)} \mu_A(z) & \text{if } h^{-1}(y) = \{x \mid h(x) = y\} \neq \phi \\ 0 & \text{otherwise} \end{cases}$$

and

$$\beta_{h(A)}(y) = \begin{cases} \inf\limits_{z\in h^{-1}(y)} \beta_A(z) & \text{if } h^{-1}(y) = \{x \mid h(x) = y\} \neq \phi \\ 0 & \text{otherwise} \end{cases}$$

(ii) Let $B = \{\langle x, \mu_B(x), \beta_B(x)\rangle \mid x \in X\}$ be an intuitionistic fuzzy set in Y. The pre-image of B under h, symbolized by $h^{-1}(B) = \{\langle x, \mu_{h^{-1}(B)}(x), \beta_{h^{-1}(B)}(x)\rangle \mid x \in X\}$ defined by: $\mu_{h^{-1}(B)}(x) = \mu_B(h(x))$ and $\beta_{h^{-1}(B)}(x) = \beta_B(h(x))$ for all $x \in X$ is an intuitionistic fuzzy set of X.

Definition 2.13[1]: Let $A = (\mu_A, \beta_A)$ and $B = (\mu_B, \beta_B)$ be any two intuitionistic fuzzy sets of a nonempty set X. The Cartesian product $A \times B$ is given by $A \times B = (\mu_{A\times B}, \beta_{A\times B})$ with membership function $\mu_{A\times B} : X \times X \to [0,1]$ and the non-membership function $\beta_{A\times B} : X \times X \to [0,1]$ are defined by $\mu_{A\times B}(x, y) = \min\{\mu_A(x), \mu_B(y)\}$ and $\beta_{A\times B}(x, y) = \max\{\beta_A(x), \beta_B(y)\}$ for all $x, y \in X$.

3. INTUITIONISTIC FUZZY H-IDEALS IN Z-ALGEBRAS:

Definition 3.1: An intuitionistic fuzzy set $A = (\mu_A, \beta_A)$ in a Z-algebra $(X, *, 0)$ is called intuitionistic fuzzy H-ideal of X if it satisfies the following conditions:

(i) $\mu_A(0) \geq \mu_A(x)$ and $\beta_A(0) \leq \beta_A(x)$

(ii) $\mu_A(x * z) \geq \min\{\mu_A(x * (y * z)), \mu_A(y)\}$

(iii) $\beta_A(x*z) \le \max\{\beta_A(x*(y*z)), \beta_A(y)\}$, for all $x, y, z \in X$

Example 3.2: Let X= {0, 1, 2, 3} be a set with the following Cayley table:

*	0	1	2	3
0	0	1	2	3
1	0	1	3	3
2	0	3	2	2
3	0	3	2	3

Then $(X,*,0)$ is a Z-algebra. Define an intuitionistic fuzzy set $A = (\mu_A, \beta_A)$ in X as follows:

$$\mu_A(x) = \begin{cases} 0.6 & \text{if} \quad x = 0 \\ 0.2 & \text{if} \quad x = 1,2,3 \end{cases} \quad \text{and} \quad \beta_A(x) = \begin{cases} 0.3 & \text{if} \quad x = 0 \\ 0.7 & \text{if} \quad x = 1,2,3 \end{cases}$$

Then A is an intuitionistic fuzzy H-ideal of a Z-algebra X.

Theorem 3.3: Intersection of any two intuitionistic fuzzy H-ideals of a Z-algebra X is again an intuitionistic fuzzy H-ideal of X.

Proof : For every $x, y, z \in X$

$$\mu_{A\cap B}(0) = \min\{\mu_A(0), \mu_B(0)\} \ge \min\{\mu_A(x), \mu_B(x)\} = \mu_{A\cap B}(x)$$

$$\beta_{A\cup B}(0) = \max\{\beta_A(0), \beta_B(0)\} \le \max\{\beta_A(x), \beta_B(x)\} = \beta_{A\cup B}(x)$$

$$\begin{aligned} \mu_{A\cap B}(x*z) &= \min\{\mu_A(x*z), \mu_B(x*z)\} \\ &\ge \min\{\min\{\mu_A(x*(y*z)), \mu_A(y)\}, \min\{\mu_B(x*(y*z)), \mu_B(y)\}\} \\ &= \min\{\mu_{A\cap B}(x*(y*z)), \mu_{A\cap B}(y)\} \end{aligned}$$

$$\begin{aligned} \beta_{A\cup B}(x*z) &= \max\{\beta_A(x*z), \beta_B(x*z)\} \\ &\le \max\{\max\{\beta_A(x*(y*z)), \beta_A(y)\}, \max\{\beta_B(x*(y*z)), \beta_B(y)\}\} \\ &= \max\{\beta_{A\cup B}(x*(y*z)), \beta_{A\cup B}(y)\} \end{aligned}$$

Hence $A \cap B$ is an intuitionistic fuzzy H-ideal of a Z-algebra X.

We generalize the above theorem as follows.

Theorem 3.4: Let $\{A_i \mid i \in \Omega\}$ be a family of intuitionistic fuzzy H-ideals of a Z-algebra X. Then $\bigcap_{i\in\Omega} A_i$ is an intuitionistic fuzzy H-Ideal of X.

By using the definition of A^c, we can prove the following result

Lemma 3.5: An IFS $A = (\mu_A, \beta_A)$ is an intuitionistic fuzzy H-ideal of a Z-algebra X if and only if the fuzzy sets μ_A and $(\beta_A)^c$ are fuzzy H-ideals of X.

Theorem 3.6: Let $A = (\mu_A, \beta_A)$ be an IFS in a Z-algebra X. Then $A = (\mu_A, \beta_A)$ is an intuitionistic fuzzy H-ideal of X if and only if $\oplus A = (\mu_A, (\mu_A)^c)$ and $\otimes A = ((\beta_A)^c, \beta_A)$ are intuitionistic fuzzy H-ideals of X.

Proof: Let $A = (\mu_A, \beta_A)$ be an intuitionistic fuzzy H-ideal of a Z-algebra X.

Let $x, y, z \in X$. Then,

(i) $\mu_A(0) \ge \mu_A(x)$ and $\mu_A(x*z) \ge \min\{\mu_A(x*(y*z)), \mu_A(y)\}$

(ii) $\beta_A(0) \le \beta_A(x)$ and $\beta_A(x*z) \le \max\{\beta_A(x*(y*z)), \beta_A(y)\}$

(iii) $(\mu_A)^c(0) = 1-\mu_A(0) \le 1-\mu_A(x) \;\; = (\mu_A)^c(x)$

(iv) $(\mu_A)^c(x*z) = 1-\mu_A(x*z) \le 1-\min\{\mu_A(x*(y*z)), \mu_A(y)\}$

$$= \max\{1-\mu_A(x*(y*z)), 1-\mu_A(y)\}$$

$$= \max\{(\mu_A)^c(x*(y*z)), (\mu_A)^c(y)\}$$

(v) $(\beta_A)^c(0) = 1-\beta_A(0) \ge 1-\beta_A(x) \;\; = (\beta_A)^c(x)$

(vi) $(\beta_A)^c(x*z) = 1-\beta_A(x*z) \ge 1-\max\{\beta_A(x*(y*z)), \beta_A(y)\}$

$$= \min\{1-\beta_A(x*(y*z)), 1-\beta_A(y)\}$$

$$= \min\{(\beta_A)^c(x*(y*z)), (\beta_A)^c(y)\}$$

From (i), (iii) and (iv) we get $\oplus A = (\mu_A, (\mu_A)^c)$ is an intuitionistic fuzzy H-ideal of a Z-algebra X.

And, from (ii), (v) and (vi) we get $\otimes A = ((\beta_A)^c, \beta_A)$ is an intuitionistic fuzzy H-ideal of a Z-algebra X.

Conversely, assume that $\oplus A = (\mu_A, (\mu_A)^c)$ and $\otimes A = ((\beta_A)^c, \beta_A)$ are intuitionistic fuzzy H-ideals of a Z-algebra X. For any $x, y, z \in X$,

(i) $\mu_A(0) \ge \mu_A(x)$ and $\beta_A(0) \le \beta_A(x)$

(ii) $\mu_A(x*z) \ge \min\{\mu_A(x*(y*z)), \mu_A(y)\}$ and

$\beta_A(x*z) \le \max\{\beta_A(x*(y*z)), \beta_A(y)\}$

Hence $A = (\mu_A, \beta_A)$ is an intuitionistic fuzzy H-ideal of a Z-algebra X.

Analogously, we can prove the following result.

Theorem 3.7: An IFS $A = (\mu_A, \beta_A)$ is an intuitionistic fuzzy H-ideal of a Z-algebra X if and only if for all $s, t \in [0,1]$, the sets $U(\mu_A; s)$ and $L(\beta_A; t)$ are either empty or H-ideals of X.

Theorem 3.8: Let h be a homomorphism from a Z-algebra $(X, *, 0)$ onto a Z-algebra $(Y, *', 0')$ and A be an intuitionistic fuzzy H-ideal of X with sup-inf property. Then image of A, $h(A) = \{\langle y, \mu_{h(A)}(y), \beta_{h(A)}(y)\rangle \mid y \in Y\}$ is an intuitionistic fuzzy H-ideal of Y.

Proof: Let $a, b, c \in Y$ with $x_0 \in h^{-1}(a)$, $y_0 \in h^{-1}(b)$ and $z_0 \in h^{-1}(c)$ such that

$$\mu_A(x_0) = \sup_{t\in h^{-1}(a)} \mu_A(t) \; ; \; \mu_A(y_0) = \sup_{t\in h^{-1}(b)} \mu_A(t) \; \text{ and } \; \mu_A(z_0) = \sup_{t\in h^{-1}(c)} \mu_A(t)$$

$$\beta_A(x_0) = \inf_{t\in h^{-1}(a)} \beta_A(t) \; ; \; \beta_A(y_0) = \inf_{t\in h^{-1}(b)} \beta_A(t) \; \text{ and } \; \beta_A(z_0) = \inf_{t\in h^{-1}(c)} \beta_A(t)$$

Now,

(i) $\mu_{h(A)}(0') = \sup_{t\in h^{-1}(0')} \mu_A(t) \ge \mu_A(0) \ge \mu_A(x_0) = \sup_{t\in h^{-1}(a)} \mu_A(t) = \mu_{h(A)}(a)$

(ii) $\beta_{h(A)}(0') = \inf_{t\in h^{-1}(0')} \beta_A(t) \le \beta_A(0) \le \beta_A(x_0) = \inf_{t\in h^{-1}(a)} \beta_A(t) = \beta_{h(A)}(a)$

(iii) $\min\{\mu_{h(A)}(a *' (b *' c)), \mu_{h(A)}(b)\} = \min\left\{ \sup_{t\in h^{-1}(a*'(b*'c))} \mu_A(t), \sup_{t\in h^{-1}(b)} \mu_A(t) \right\}$

$$\leq \min\{\mu_A(x_0 * (y_0 * z_0)), \mu_A(y_0)\}$$
$$\leq \mu_A(x_0 * z_0)$$
$$= \sup_{t \in h^{-1}(a *' c)} \mu_A(t) = \mu_{h(A)}(a *' c)$$

(iv) $\max\{\beta_{h(A)}(a *' (b *' c)), \beta_{h(A)}(b)\} = \max\left\{ \inf_{t \in h^{-1}(a *' (b *' c))} \beta_A(t), \inf_{t \in h^{-1}(b)} \beta_A(t) \right\}$

$$\geq \max\{\beta_A(x_0 * (y_0 * z_0)), \beta_A(y_0)\}$$
$$\geq \beta_A(x_0 * z_0)$$
$$= \inf_{t \in h^{-1}(a *' c)} \beta_A(t) = \beta_{h(A)}(a *' c)$$

Hence h(A) is an intuitionistic fuzzy H- ideal of a Z-algebra Y.

Theorem 3.9: Let $h : (X,*,0) \to (Y,*',0')$ be a Z-homomorphism of Z-algebras and B be an intuitionistic fuzzy H-ideal of a Z-algebra Y. Then the inverse image of B, $h^{-1}(B) = \{\langle x, \mu_{h^{-1}(B)}(x), \beta_{h^{-1}(B)}(x)\rangle | x \in X\}$ is an intuitionistic fuzzy H-ideal of a Z-algebra X.

Proof: Let $x, y, z \in X$. Now it is clear that

$$\mu_{h^{-1}(B)}(0) = \mu_B(h(0)) = \mu_B(0') \geq \mu_B(h(x)) = \mu_{h^{-1}(B)}(x)$$
$$\beta_{h^{-1}(B)}(0) = \beta_B(h(0)) = \beta_B(0') \leq \beta_B(h(x)) = \beta_{h^{-1}(B)}(x)$$
$$\mu_{h^{-1}(B)}(x * z) = \mu_B(h(x * z)) = \mu_B(h(x) *' h(z)) \geq \min\{\mu_B(h(x) *' (h(y) *' h(z))), \mu_B(h(y))\}$$
$$= \min\{\mu_B(h(x * (y * z))), \mu_B(h(y))\}$$
$$= \min\{\mu_{h^{-1}(B)}(x * (y * z)), \mu_{h^{-1}(B)}(y)\}$$

$$\beta_{h^{-1}(B)}(x * z) = \beta_B(h(x * z)) = \beta_B(h(x) *' h(z)) \leq \max\{\beta_B(h(x) *' (h(y) *' h(z))), \beta_B(h(y))\}$$
$$= \max\{\beta_B(h(x * (y * z))), \beta_B(h(y))\}$$
$$= \max\{\beta_{h^{-1}(B)}(x * (y * z)), \beta_{h^{-1}(B)}(y)\}$$

Hence $h^{-1}(B)$ is an intuitionistic fuzzy H-ideal of a Z-algebra X.

Analogously, we can prove the following result.

Theorem 3.10: Let $h : (X,*,0) \to (Y,*',0')$ be an Z-epimorphism of Z-algebras. Let B be an intuitionistic fuzzy set of a Z-algebra Y. If $h^{-1}(B)$ is an intuitionistic fuzzy H-ideal of a Z-algebra X then B is an intuitionistic fuzzy H-ideal of a Z-algebra Y.

Theorem 3.11: Let h be an Z-endomorphism of Z-algebra $(X,*,0)$. If A be an intuitionistic fuzzy H-ideal of X. Then the intuitionistic fuzzy set $A^h = (\mu_{A^h}, \beta_{A^h})$ is also an intuitionistic fuzzy H-ideal of X.

Proof: Follows directly from the definition.

Theorem 3.12: Let A and B be two intuitionistic fuzzy H-ideals in a Z-algebra X. Then $A \times B$ is an intuitionistic fuzzy H-ideal of $X \times X$.

Proof: Take $(x_1, x_2) \in X \times X$.

Then $\mu_{A\times B}(0,0) = \min\{\mu_A(0), \mu_B(0)\} \geq \min\{\mu_A(x_1), \mu_B(x_2)\} = \mu_{A\times B}(x_1, x_2)$

and $\beta_{A\times B}(0,0) = \max\{\beta_A(0), \beta_B(0)\} \leq \max\{\beta_A(x_1), \beta_B(x_2)\} = \beta_{A\times B}(x_1, x_2)$

Now take $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$. Then

$$\mu_{A\times B}(x_1 * z_1, x_2 * z_2) = \min\{\mu_A(x_1 * z_1), \mu_B(x_2 * z_2)\}$$

$$\geq \min\{\min\{\mu_A(x_1 * (y_1 * z_1)), \mu_A(y_1)\}, \min\{\mu_B(x_2 * (y_2 * z_2)), \mu_B(y_2)\}\}$$

$$= \min\{\min\{\mu_A(x_1 * (y_1 * z_1)), \mu_B(x_2 * (y_2 * z_2))\}, \min\{\mu_A(y_1), \mu_B(y_2)\}\}$$

$$= \min\{\mu_{A\times B}((x_1, x_2) * ((y_1, y_2) * (z_1, z_2))), \mu_{A\times B}(y_1, y_2)\}$$

$$\beta_{A\times B}(x_1 * z_1, x_2 * z_2) = \max\{\beta_A(x_1 * z_1), \beta_B(x_2 * z_2)\}$$

$$\leq \max\{\max\{\beta_A(x_1 * (y_1 * z_1)), \beta_A(y_1)\}, \max\{\beta_B(x_2 * (y_2 * z_2)), \beta_B(y_2)\}\}$$

$$= \max\{\max\{\beta_A(x_1 * (y_1 * z_1)), \beta_B(x_2 * (y_2 * z_2))\}, \max\{\beta_A(y_1), \beta_B(y_2)\}\}$$

$$= \max\{\beta_{A\times B}((x_1, x_2) * ((y_1, y_2) * (z_1, z_2))), \beta_{A\times B}(y_1, y_2)\}$$

Hence $A \times B$ is an intuitionistic fuzzy H-ideal of $X \times X$.

Analogously, we can prove the following results.

Theorem 3.13: Let A and B be two intuitionistic fuzzy sets of a Z-algebra X. If $A \times B$ is an intuitionistic fuzzy H-ideal of $X \times X$, the following are true.

1. $\mu_A(0) \geq \mu_B(y)$ and $\mu_B(0) \geq \mu_A(x)$ for all $x, y \in X$.
2. $\beta_A(0) \leq \beta_B(y)$ and $\beta_B(0) \leq \beta_A(x)$ for all $x, y \in X$.

Theorem 3.14: Let A and B be two intuitionistic fuzzy sets of a Z-algebra X such that $A \times B$ is an intuitionistic fuzzy H-ideal of $X \times X$. Then either A or B is an intuitionistic fuzzy H-Ideal of X.

4. CONCLUSION:

In this paper, intuitionistic fuzzy H-ideals in Z-algebras is introduced and investigated some of their useful properties. In our future study of fuzzy structure of Z-algebras, may be the following topics should be considered: (i) to find translation of intuitionistic fuzzy H-ideals in Z-algebras, (ii) to find multiplication of intuitionistic fuzzy H-ideals in Z-algebras.

REFERENCES:

[1] K.T. Atanassov: Intuitionistic Fuzzy Sets, Fuzzy Sets and Systems, 20(1)(1986), pp.87-96.

[2] K.T. Atanassov: More on Intuitionistic Fuzzy sets, Fuzzy Sets and Systems, 33(1)(1989), pp.37-45.

[3] M. Chandramouleeswaran, P. Muralikrishna, K. Sujatha and S. Sabarinathan: A note on Z-algebra, Ital. J. Pure Appl. Math., 38 (2017), pp.707-714.

[4] Y. Imai and K. Iseki, On axiom systems of propositional calculi XIV, Proceedings of the Japan Academy 42 (1966), pp.19-22.

[5] K. Iseki, An algebra related with a propositional calculus, Proceedings of the Japan Academy 42 (1966), pp.26-29.

[6] X.P. Li and G.J. Wang: Intuitionistic fuzzy group and its homomorphic image, Fuzzy Syst. Math., 14(1)(2000), No. 1, pp.45-50.

[7] S. Sowmiya and P. Jeyalakshmi: On fuzzy H-ideals in Z-algebras, Advances in Fuzzy Sets and Systems, 27(2022), No. 2, pp.243-251.

[8] L. A. Zadeh, Fuzzy sets, Information and Control 8(3) (1965), pp.338-353.

STBE ALGEBRAS – CONSTRUCTED FROM IDEALS

P. Lakshmi Kumari[1] and V. Thiruveni[2]

[1] Research Scholar, PG and Research Department of Mathematics,
Saiva Bhanu Kshatriya College, (Affiliated to Madurai Kamaraj University, Madurai)
Aruppukottai, , Tamil Nadu, India. malathyselvaraj77@gmail.com

[2] Assistant Professor, PG and Research Department of Mathematics,
Saiva Bhanu Kshatriya College, (Affiliated to Madurai Kamaraj University, Madurai)
Aruppukottai, Tamil Nadu, India. thiriveni2009@gmail.com

ABSTRACT : *In this paper, the role of ideals of STBE-algebras is discussed. Using the ideals, we construct an STBE-algebra and analyze the completion of the topologies that we have constructed. Results analogues to that of topological rings are also derived.*

Keywords: *STBE-Algebra, Ideals in STBE-Algebras, Directed sets, Quotient topology, Inverse system.*

1. INTRODUCTION:

To compare set theory with the logical systems, Y.Imai and K.Iseki introduced a new classes of algebras, called BCK-algebras and BCH-algebras. Many authors investigated these algebras. In [2], H.A.Kim and Y.H. Kim introduced the notion of BE-algebras, which is the generalization of BCK-algebras. In [7], Jansi M and Thiruveni V introduced the notion of ideals in TSBF-algebras. In [5], Thiruveni V, Lakshmi kumara P and Latha K.B studied separation axioms on S-Topological BE-algebras. In this paper, we discuss the role of ideals of STBE-algebras (S-Topological BE-algebras) and we construct STBE-algebras using ideals.

2. PRELIMINARIES:

Definition 2.1 [2] A BE-algebra is an algebra (X,*,1) of type (2,0) (that is, a non-empty set X with a binary operation * and a constant 1) satisfying the following conditions

1. $x * x = 1$
2. $x * 1 = 1$
3. $1 * x = x$
4. $x * (y * z) = y * (x * z),\ \forall\, x, y, z\ \in X.$

Definition 2.2 [2] A BE-algebra (X,*,1) is called a commutative BE-algebra if it satisfies the identity$(x * y) * y = (y * x) * x, \forall\, x, y\ \in X.$

Theorem 2.3 [2] If X is a commutative BE-algebra then $x * y = 1$ or $y * x = 1,$ for all distinct $x, y\ \in X.$

Definition 2.4 [3] A subset A of a topological space is said to be semi-open if $\subseteq \overline{Int\ A}$.

Definition 2.5 [3] The complement of a semi-open set is called semi-closed.

Definition 2.6 [3] The semi-closure of a subset A of a topological space is the intersection of all semi-closed set containing A. It is denoted by $\bar{A}^S$.

Definition 2.7 [3] A subset A of a topological space is said to be regular open if $= \overline{Int\ A}$.

Definition 2.8 [5] A BE-algebra $(X,*,1)$ equipped with a topology τ_S is called S-topological BE-algebra (STBE-algebra) is the function $f: X \times X \to X$ defined by, $f(x,y) = x * y$ has the property that for each open set O containing $x * y$, there exists a open set U containing x and a semi-open set V containing y such that, $U * V \subseteq O$, for all $x, y \in X$.
Definition 2.9 [6] Let $(X,*,\tau_S)$ be a STBE-algebra. A non-empty subset $A \subseteq X$ is called an ideal of X if

1) $1 \in A$,
2) $\forall y \in X$ and $\forall x \in A$, if $x * y \in A$, then $y \in A$.

Definition 2.10 [8] Let S be a partially ordered set. S is called a directed set if for $i, j \in S, \exists\, k \in S$, such that $i \leq k$ and $j \leq k$.
Definition 2.11 [8] Let $I \neq \varphi$ be a subset of a BE-algebra X. Define a binary relation $\equiv (mod\ I)$ as follows:
$x \equiv y\ (mod\ I)$ if $x * y \in I$ and $y * x \in I$. The set $\{b \in X/b \equiv a\ mod\ I\}$ is denoted by $[a]_I$.

3. ROLE OF IDEALS IN BE-ALGEBRAS:
Definition 2.1 Let $(X,*,1)$ be a BE-algebra and S be a directed set. Define the family of ideals of X as $\mathcal{F} = \{I_k/k \in S\}$ such that $I_k \supset I_{k,}\ if\ i < j$. ---------------- (1)
Remark 2.2: For $a \in X, k \in S$, define $U(a,k) = \{x \in X/x \equiv a(mod\ I_K)\}$.
Then $\tau_k = \{U(a,k)/k \in S\} \cup \varphi$ is a topology on X. Also $\{I_k/k \in S\}$ is a topology on X.
Remark 2.3: 1. Fix $a \in X$ and $k \in S$.
Then we have $U(a,k) = X - \cup\, \{U(a,k)/\ x \not\equiv a\ mod\ I_k\}$. So, $U(a,k)$ is both open and closed.
Theorem 2.4 Let $(X,*,1)$ be a STBE-algebra. Suppose that {1} is closed (open). Then {a} is closed (open) for all $a \in X$.
Proof: Let $(X,*,1)$ be a STBE-algebra. Then $f: XxX \to X$ be the continuous map defined by $f(a,b) = a * b$.
Now, we define a map $g: XxXxX \dashrightarrow XxX$ by $g(a,b,c) = (a * b, b * c)$.
As f is continuous, g is continuous.
Suppose that {1} is closed. Then {1,1} is closed in XxX.
Fix $a \in X$. Define a map $h: X \dashrightarrow XxX$ by $h(b) = g(a,b,a) = (a * b, b * a)$. Then h is the restriction of g to $\{a\}xXx\{a\}$. So, h is continuous.
Now, $h^{-1}\ (1,1) = \{b/a * b = 1\ and\ b * a = 1\} = \{a\} \Rightarrow \{a\}$ is closed (open) as {1} is closed (open).
Now, we construct the inverse system.
The quotient topology $X/_{I_k}$ on each I_k is discrete. If $i < j$, there is a natural homomorphism $\varphi_{ij}: X/_{I_i} \to X/_{I_j}$. So, we can construct the inverse system $\{X/_{I_i}, \varphi_{ij}\}$. The inverse limit is $\lim\limits_{\leftarrow} X/_{I_i} = \hat{X}$. Then $\hat{X}$ is the completion of X. The following lemma is obvious.
Lemma 2.5 $\psi : X \to \hat{X}$ is continuous and $\psi\ (X)$ is dense in $\hat{X}$.
Remark 2.6 Let $\pi_i: \hat{X} \to X/_{I_i}$. Then π_i is a natural projection. Let $X^* = \ker \pi_i$
Then $\{X_i^*/i \in S\}$ is a family of ideals of X^* such that if $i < j$, then $X_j^* \subset X_i^*$
Now, for each $i \in S$ and each $\{[a_k]/k \in S\} \in \hat{X}$.

Let $\cup\,(\{[a_k]\}_{k\in S}\,, i) = \{\{[b_k]\}_{k\in S} \in \hat{X}\,/\{[b_k]\}_{k\in S} \equiv \{[a_k]\}_{k\in S}\, mod\ X_i^*$.
Then$\cup\,\big(\{[a_{k]}\}_{k\in S}\,, i\big) = \prod\{U_k/k \in S\}$, where $U_k = \pi_k(\widehat{X})\ if\ k \neq i$ and U_k is the singleton $[a_i]$. Hence $\cup\,\big(\{[a_{k]}\}_{k\in S}\,, i\big)$ is open. So, we get a topology induced by the family of ideals $\{X_i^*/i \in S\}$.
Also we see that $\pi_i\big(\psi(X)\big) = {}^{X}/_{I_i} \Rightarrow \pi_i$ is onto.
Hence ${}^{X}/_{I_i} \cong {}^{\hat{X}}/_{\ker \pi_i} = {}^{\hat{X}}/_{X_i^*}$. So, the completion of $\hat{X}$ is $\hat{X}$.
Consider two directed sets S_1 and S_1. Then the sets of ideals $\mathcal{F}_1 = \{I_i\,/i \in S_1\}$ and $\mathcal{F}_2 = \{J_i\,/i \in S_2\}$ both induce the same topology on X if and only if for each $U(a,i)$ $(i \in S_1)$, there exists $k \in S_2$ such that $U(a,k) \subset U(a,i)$ and for each $U(a,k)$ $(k \in S_2$ there exist $i \in S_1$ such that $U(a,i) \subset U(a,k)$. If this is the case, then $\varprojlim {}^{X}/_{I_i} \longrightarrow \varprojlim {}^{X}/_{I_i}$ is an isomorphism.
Now, the following lemma is obvious.
Lemma 2.7 $\psi : X \to \hat{X}$ is injective if and only is $\cap\,\{I_i/i \in S\} = \{0\}$. That is if and only if X is T_1.
Theorem 2.8 If $A \subset X$, then $\tilde{A} = \bigcap_{i\in S} \bigcup_{x\in A} U(x,i)$, where $\tilde{A}$ is the topological closure of A.
Proof: Let $A \subset X$. We have $X - \cup\,\{U(x,i)/x \in A\} = \cup\,\{U(y,i)/\ for\ all\ x \in A, y \not\equiv x\}$
Since $\cup\,\{U(y,i)/\ for\ all\ x \in A, y \not\equiv x\}$ is open, we have $X - \cup\,\{U(x,i)/x \in A\}$ is open.
$\Rightarrow \cup\,\{U(x,i)/x \in A\}$ is closed and it contains A. $\Rightarrow\ \tilde{A} = \bigcap_{i\in S} \bigcup_{x\in A} U(x,i)$,
Theorem 2.9 If I is an open (closed) ideal, then for each $x \in X, \{y \in X/y \equiv x\ mod\ I\}$ is open (closed).
Proof: Let $x \in X$. Define a left map $L_x\colon X \to X$, by $L_x(y) = x * y$ and a right map $R_x\colon X \to X$, by $R_x(y) = y * x$.Then L_x and R_x are continuous.
Assume that I is an open ideal. $\Rightarrow$ $L_x^{-1}(I)$ and $R_x^{-1}(I)$ are open $\Rightarrow$ $L_x^{-1}(I) \cap R_x^{-1}(I)$ is open.
But $L_x^{-1}(I) \cap R_x^{-1}(I) = \{y \in X/x * y, y * x \in I\} = \{y \in X/y \equiv x\ mod\ I\}$.
Hence, $\{y \in X/y \equiv x\ mod\ I\}$ is open.
Similarly, we can prove that if I is closed, then $\{y \in X/y \equiv x\ mod\ I\}$ is closed.
Theorem 2.10 Every open ideal in X is closed.
Proof: Let I be an open ideal of X. From theorem 2.9, for each $x \in X$, $\{y \in X/y \equiv x\ mod\ I\}$ is open. But I is the complement of the union of all other congruence classes.
That is $= \{\cup\,\{y \in X/y \not\equiv x\ mod\ I\}\}^C$.
Since, $\cup\,\{y \in X/y \not\equiv x\ mod\ I\}$ is open, its complement I is closed.

REFERENCES :

[1] H.S. Kim and Y.H. Kim, On BE-algebras, *Scientiae Mathematicae Japonicae 66 (2007),* 113-128.
[2] Aldrzej Walendziak, On Commutative BE-algebras, *Scientiae Mathematicae Japonicae (e-2008),* 585-588.
[3] Levine N, Semi-open sets and semi-continuity in topological spaces, *American Mathematical Monthly 70 (1963),* 36-41.
[4] S.N.Maheswari and R. Prasad, Some new separation axioms, *Annales de la Socite Scientifique de Bruexles, T.,89III(1975),* 395-402.
[5] P. Lakshmi Kumari and V. Thiruveni, S-Topological BE-Algebras, *Advances and*

Applications in Mathematical Sciences, Accepted.
[6] S. Mehrshad and J. Golzarpoor, On topological BE-Algebras, *Mathematica Moravica, Vol.21, No.2(2017),*1-13.
[7] M. Jansi and V. Thiruveni, Complementary role of ideals in TSBF-Algebras, *Malaya Journal of Mathematik, Vol.8, No.3,* 1037-1040, 2020.
[8] Yong Ho Yon, Sang Moon Lee, Kyung Ho Kim, On Congruences and BE-relations in BE-algebras, *International Mathematical Forum*, 5,2010, No.46, 2263-2270.

k – INTUITIONISTICS FUZZY IDEALS

P. Suseela

Assistant Professor, PG and Research department of Mathematics,
Saiva Bhanu Kshatriya college,
Aruppukottai-626101, Tamil Nad, India.
E-mail : suceela93@gmail.com

ABSTRACT : *A more natural and necessary generalization of the intuitionistic fuzzy theory is developed and introduce the concept of k-intuitionistic fuzzy ideals. In this paper we prove many theorems in the concept of ideals.*

1. INTRODUCTION:

In the current century fuzzy theory has its vast applications in almost all fields, which are related to Mathematics technically. Based on the concept of fuzzy theory, intuitionistic fuzzy theory was develop, but the theory in some way didn't emerged as much as fuzzy theory. In most natural situations like buying a new car, judging about a persons various personalities, both fuzzy theory and intuitionistic fuzzy theory were insufficient. So, we are in need to develop a new structure to annihilate the insufficiency.

Fuzzy subsets were developed by Zadeh [7] as functions from a set X to the closed interval $[0,1] \subseteq \mathbb{R}$ to study the uncertainties; it study the gradual membership of an object in a set. In the name Zadeh, fuzzy theory has emerged as an important notion in the field of Mathematics. Many of its branches, like fuzzy group theory, fuzzy topology, fuzzy metricspaces were developed and studied by many others. Joseph G. Brown, A. Rosenfeld, W.M. Wu, Rajeshkumar, [4] are some, who studied fuzzy theory in the context of Algebra. K. T. Atanassov [1] developed the concept intuitionistic fuzzy subsets in 1983.

Basic definitions:

Definition: 1.1

Let S be any nonempty set. A mapping $\mu: S \rightarrow [0,1]$ is called a fuzzy subset of S.

Definition: 1.2

Let μ be any fuzzy subset of a set S and let $t \in [0,1]$. The set $\mu_t = \{x \epsilon S / \mu(x) \geq t\}$ is called a level subset of μ.

Definition: 1.3

Let f be any function from a set S to a set T. Let μ be any fuzzy subset of S and let σ be any fuzzy subset of T. Then the image of μ under f denoted by $f(\mu)$, is a fuzzy subset of T denoted by:

$$\big(f(\mu)\big)(y) = \begin{cases} \sup\limits_{x\epsilon f^{-1}(x)} \mu(x) & if\ f^{-1}(x) \neq \emptyset \\ 0 & otherwise \end{cases}$$

where $y \in T$.

The inverse image of σ under f, symbolized as $f^{-1}(\sigma)$, is a fuzzy subset of S, defined by $f^{-1}(\sigma)(x) = \sigma(f(x))$ for all $x \epsilon S$.

Definition :1.4

A fuzzy subset μ of a ring $\mathbb{R}$ is called a fuzzy subring of $\mathbb{R}$, if

$$(i)\ \mu(x-y) \geq \mu(x) \wedge \mu(y)$$
$$(ii)\ \mu(xy) \geq \mu(x) \wedge \mu(y) \quad \text{for all } x, y \epsilon \mathbb{R}$$

Definition : 1.5

A fuzzy subset μ of a ring $\mathbb{R}$ is called a fuzzy ideal of $\mathbb{R}$, if

$$(i)\ \mu(x-y) \geq \mu(x) \wedge \mu(y)$$
$$(ii)\ \mu(xy) \geq \mu(x) \vee \mu(y) \quad \text{for all } x, y \epsilon \mathbb{R}$$

Definition: 1.6

Let X be a fixed non-empty set. An intuitionistic fuzzy set A in X is an object having the form $A^* = \{\langle x, \mu_A(x), \vartheta_A(x)\rangle / x \epsilon X\}$, where $\mu_A : X \to [0,1]$ and $\vartheta_A : X \to [0,1]$ define the degree of membership and degree of non-membership of the element $x \epsilon X$ to the set A, which is a subset of X, respectively, and for every $x \epsilon X$, we have $0 \leq \mu_A(x) + \vartheta_A(x) \leq 1$

Definition: 1.7

For any two intuitionistic fuzzy subsets A and B of a set X, the following properties hold:

- $A \subset B\ if\ \mu_A(x) \leq \mu_B(x)$ and $\vartheta_A(x) \geq \vartheta_B(x)$, for all $x \epsilon X$.
- $A = B\ if A \subset B$ and $B \subset A$
- $\bar{A} = \{\langle x, \mu_A(x), \vartheta_A(x)\rangle\}, x \epsilon X$
- $A \cap B = \{\langle x,\ \min\{\mu_A(x), \mu_B(x)\},\ \max\{\vartheta_A(x), \vartheta_B(x)\}\rangle\}, x \epsilon X$
- $A \cup B = \{\langle x,\ \max\{\mu_A(x), \mu_B(x)\},\ \min\{\vartheta_A(x), \vartheta_B(x)\}\rangle\}, x \epsilon X$

Note: In what follows in this paper, we define an intuitionistic fuzzy set A of set X as a pair (μ_A, ϑ_A), for simplicity.

2 k-INTUITIONISTIC FUZZY STRUCTURES:

In our trending world, fuzzy theory has its wide applications in almost all fields, for example, signal processing, telecommunication, aerospace, automotive, robotics, chemical industry, electronics, medical, mining and metal processing. Even though fuzzy logic has emerged as unavoidable branch of mathematics, the theory is insufficient in some sense in many real life situations. For example, while buying a plot in a city, as a buyer one man will have his own desire and expectation about his plot. That is, he may expect, the plot should be around 3000sqft, the ground water level should be high; bus stand, schools, hospitals and colleges should be at minimum distance; there should be a good road facility, etc., It is not quite possible practically, to fulfill all his expectations. So, if a buyer say I will buy a plot, only if all my expectations are fulfilled means; he will never buy a plot in his lifetime. So, he should relax his own level of expectations; matter of acceptance and opposition level of his expectations, plays a vital role here. Thus we need a structure, to discuss about the level of acceptance and the level of opposition, of a finite set of properties of an object.

Definition : 2.1

Let X be a non-empty set. Let k be a positive integer. Then a k-intuitionistic fuzzy subset of a set X is an ordered $2k$ tuple $(\mu_1, \mu_2, \dots \mu_k, \vartheta_1, \vartheta_2, \dots \vartheta_k)$ of functions from $X$to $[0,1]$ satisfying $\mu_i(x) + \vartheta_i(x) \leq 1\ for\ all\ i = 1,2, \dots, k\ and\ for\ all\ x \in X.$

We denote a k- intuitionistic fuzzy subset A as an ordered $2k$ tuple $(\mu_{A_1}, \mu_{A_2}, \dots \mu_{A_k}, \vartheta_{A_1}, \vartheta_{A_2}, \dots \vartheta_{A_k})$ throughout the paper.

Definition: 2.2

For any two k-intuitionistic fuzzy subsets A and B of a set X, we define

- $A \subseteq B \; if \; \mu_{A_i}(x) \le \mu_{B_i}(x)$ and $\vartheta_{A_i}(x) \ge \vartheta_{B_i}(x)$, for all $x \epsilon X$ and $for \; all \; i = 1,2,\dots,k$
- $A = B \; if A \subseteq B$ and $B \subseteq A$
- $\bar{A}(x) = (\vartheta_{A_1}(x), \vartheta_{A_2}(x), \dots \vartheta_{A_k}(x), \mu_{A_1}(x), \mu_{A_2}(x), \dots \mu_{A_k}(x))$, $x \epsilon X$
- $(A \cap B)(x) = ((\mu_{A_1}(x) \wedge \mu_{B_1}(x)), \dots, (\mu_{A_k}(x) \wedge \mu_{B_k}(x)),$
  $(\vartheta_{A_1}(x) \vee \vartheta_{B_1}(x)), \dots, (\vartheta_{A_k}(x) \vee \vartheta_{B_k}(x))$, $x \epsilon X$
- $(A \cup B)(x) = ((\mu_{A_1}(x) \wedge \mu_{B_1}(x)), \dots, (\mu_{A_k}(x) \vee \mu_{B_k}(x)),$
  $(\vartheta_{A_1}(x) \wedge \vartheta_{B_1}(x)), \dots, (\vartheta_{A_k}(x) \wedge \vartheta_{B_k}(x))$, $x \epsilon X$

Definition: 2.3

Let f be a function from X to Y and let $A = (\mu_{A_1}, \mu_{A_2}, \dots \mu_{A_k}, \vartheta_{A_1}, \vartheta_{A_2}, \dots \vartheta_{A_k})$ be a $k-$ intuitionistic fuzzy subset in X. The image of A, written as $f(A)$ is a $k-$ intuitionistic in Y is given by,

$$f(A) = (\mu_{f(A)_1}, \mu_{f(A)_2}, \dots \mu_{f(A)_k}, \vartheta_{f(A)_1}, \vartheta_{f(A)_2}, \dots \vartheta_{f(A)_k})$$

where,

$$\mu_{f(A)_i}(y) = \begin{cases} \sup\limits_{x \epsilon f^{-1}(x)} \{\mu_{A_i}(x)\} & if \; f^{-1}(y) \neq \emptyset \\ 0 & otherwise \end{cases}$$

and

$$\vartheta_{f(A)_i}(y) = \begin{cases} \inf\limits_{x \epsilon f^{-1}(x)} \{\vartheta_{A_i}(x)\} & if \; f^{-1}(y) \neq \emptyset \\ 1 & otherwise \end{cases}$$

Definition: 2.4

Let f be a function from X to Y and let $A = (\mu_{A_1}, \mu_{A_2}, \dots \mu_{A_k}, \vartheta_{A_1}, \vartheta_{A_2}, \dots \vartheta_{A_k})$ be a $k-$ intuitionistic fuzzy subset in Y. Then the inverse of A is written as $f^{-1}(A)$ is a $k-$ intuitionistic fuzzy subset in X given by,

$f^{-1}(A) = (\mu_{f^{-1}(A)_1}, \mu_{f^{-1}(A)_2}, \dots \mu_{f^{-1}(A)_k}, \vartheta_{f^{-1}(A)_1}, \vartheta_{f^{-1}(A)_2}, \dots \vartheta_{f^{-1}(A)_k})$

where $\mu_{f^{-1}(A)_i}(x) = \mu_{A_i}(f(x))$ and $\vartheta_{f^{-1}(A)_i}(x) = \vartheta_{A_i}(f(x))$, $for \; all \; i = 1,2,\dots,k \; and$ for all $x \in X$.

3 k-INTUIOTIONISTIC FUZZY IDEALS :

Definition: 3.1

Let $\mathbb{R}$ be a ring. A $k-$ intuitionistic fuzzy subset A of $\mathbb{R}$ is said to be a $k-$ intuitionistic fuzzy subring of $\mathbb{R}$ if it satisfies the following conditions:

(i) $\mu_{A_i}(x - y) \ge \mu_{A_i}(x) \wedge \mu_{A_i}(y)$

(ii) $\mu_{A_i}(xy) \ge \mu_{A_i}(x) \wedge \mu_{A_i}(y)$

(iii) $\vartheta_{A_i}(x - y) \le \vartheta_{A_i}(x) \vee \vartheta_{A_i}(y)$

(iv) $\vartheta_{A_i}(xy) \le \vartheta_{A_i}(x) \vee \vartheta_{A_i}(y) \; for \; all \; i = 1,2,\dots,k \; and$ for all $x \in X$

Definition : 3.2

Let $\mathbb{R}$ be a ring. A $k -$ intuitionistic fuzzy subset A of $\mathbb{R}$ is said to be a $k -$ intuitionistic fuzzy ideal of $\mathbb{R}$ if it satisfies the following conditions;

(i) $\mu_{A_i}(x - y) \geq \mu_{A_i}(x) \wedge \mu_{A_i}(y)$

(ii) $\mu_{A_i}(xy) \geq \mu_{A_i}(x) \vee \mu_{A_i}(y)$

(iii) $\vartheta_{A_i}(x - y) \leq \vartheta_{A_i}(x) \vee \vartheta_{A_i}(y)$

(iv) $\vartheta_{A_i}(xy) \leq \vartheta_{A_i}(x) \wedge \vartheta_{A_i}(y)\ for\ all\ i = 1,2, \dots, k\ and$ for all $x \in X$

Example: 3.3

Let $\mathbb{R}$ be a ring of real numbers under the usual operations of addition and multiplication. Then the $k -$ intuitionistic fuzzy subset A of $\mathbb{R}$ defined by

$$\mu_{A_i}(x) = \begin{cases} 0 & if\ x\ is\ rational \\ 0.8 & otherwise \end{cases}$$

$$\vartheta_{A_i}(x) = \begin{cases} 1 & if\ x\ is\ rational \\ 0.1 & otherwise \end{cases}$$

A is a $k -$ intuitionistic fuzzy ideal of $\mathbb{R}$.

Theorem : 3.4

Let $\mathbb{R}$ be a ring. Let A and B be any two $k -$ intuitionistic fuzzy ideal. Then $A \cap B$ is also a $k -$ intuitionistic fuzzy ideal.

Proof:

Let $A = (\mu_{A_1}, \mu_{A_2}, \dots \mu_{A_k}, \vartheta_{A_1}, \vartheta_{A_2}, \dots \vartheta_{A_k})$

And $B = (\mu_{B_1}, \mu_{B_2}, \dots \mu_{B_k}, \vartheta_{B_1}, \vartheta_{B_2}, \dots \vartheta_{B_k})$ be two $k -$ intuitionistic fuzzy ideal.

Then we have $(A \cap B)(x) = ((\mu_{A_1}(x) \wedge \mu_{B_1}(x)), \dots, (\mu_{A_k}(x) \wedge \mu_{B_k}(x)),$

$(\vartheta_{A_1}(x) \vee \vartheta_{B_1}(x)), \dots, (\vartheta_{A_k}(x) \vee \vartheta_{B_k}(x))$ for all $x \in X$.

We take $(\mu_{A_i} \wedge \mu_{B_i}) = \gamma_i$ and $(\vartheta_{A_1} \vee \vartheta_{B_1}) = \gamma_i'\ for\ all\ i = 1,2, \dots, k$

Then, $A \cap B = (\gamma_1, \gamma_2, \dots, \gamma_k, \gamma_1', \gamma_2', \dots, \gamma_k')$

Now, $\gamma_i(x - y) = (\mu_{A_i}(x - y) \wedge \mu_{B_i}(x - y))$

$\geq (\mu_{A_i}(x) \wedge \mu_{A_i}(y)) \wedge (\mu_{B_i}(x) \wedge \mu_{B_i}(y))$

$= (\mu_{A_i}(x) \wedge \mu_{B_i}(x)) \wedge (\mu_{A_i}(y) \wedge \mu_{B_i}(y))$

$= [\gamma_i(x) \wedge \gamma_i(y)]$

Therefore, $\gamma_i(x - y) \geq [\gamma_i(x) \wedge \gamma_i(y)]$

Now, $\gamma_i(xy) = (\mu_{A_i}(xy) \wedge \mu_{B_i}(xy))$

$\geq (\mu_{A_i}(x) \vee \mu_{A_i}(y)) \wedge (\mu_{B_i}(x) \vee \mu_{B_i}(y))$

$= (\mu_{A_i}(x) \wedge \mu_{B_i}(x)) \vee (\mu_{A_i}(y) \wedge \mu_{B_i}(y))$

$= [\gamma_i(x) \vee \gamma_i(y)]$

Therefore, $\gamma_i(xy) \geq [\gamma_i(x) \vee \gamma_i(y)]$

Similarly, we can prove (iii) and (iv) of the definition (13).

Theorem : 3.5

Let $\mathbb{R}$ and $\mathbb{R}'$ be two rings and $f: \mathbb{R} \to \mathbb{R}'$ be an onto homomorphism. Let A be a $k -$ intuitionistic fuzzy ideal of $\mathbb{R}$. Then $f(A)$ is a $k -$ intuitionistic fuzzy ideals of $\mathbb{R}'$.

Proof: Let $A = (\mu_{A_1}, \mu_{A_2}, \dots \mu_{A_k}, \vartheta_{A_1}, \vartheta_{A_2}, \dots \vartheta_{A_k})$ be a $k -$ intuitionistic fuzzy ideal of $\mathbb{R}$.

Claim: $f(A)$ is a $k -$ intuitionistic fuzzy ideals of $\mathbb{R}'$

Let $x, y \in \mathbb{R}'$, then we have

$$\mu_{f(A)_i}(x) = \begin{cases} \sup\limits_{u\epsilon f^{-1}(x)}\{\mu_{A_i}(u)\} & if\ f^{-1}(x) \neq \emptyset \\ 0 & otherwise \end{cases}$$

$$\vartheta_{f(A)_i}(x) = \begin{cases} \inf\limits_{u\epsilon f^{-1}(x)}\{\vartheta_{A_i}(u)\ \} & if\ f^{-1}(x) \neq \emptyset \\ 1 & otherwise \end{cases}$$

$$\mu_{f(A)_i}(y) = \begin{cases} \sup\limits_{v\epsilon f^{-1}(y)}\{\mu_{A_i}(v)\} & if\ f^{-1}(y) \neq \emptyset \\ 0 & otherwise \end{cases}$$

and

$$\vartheta_{f(A)_i}(y) = \begin{cases} \inf\limits_{v\epsilon f^{-1}(y)}\{\vartheta_{A_i}(v)\ \} & if\ f^{-1}(y) \neq \emptyset \\ 1 & otherwise \end{cases}$$

Now we prove that,

If $u \in f^{-1}(x)$ and $v \in f^{-1}(y)$, then $u - v \in f^{-1}(x - y)$ and $uv \in f^{-1}(xy)$

For,

Let $u \in f^{-1}(x)$ and $v \in f^{-1}(y)$ then $f(u) = x$ and $f(v) = y$

$$\Rightarrow f(u) - f(v) = f(u - v) = x - y$$

$$\Rightarrow u - v\ \in f^{-1}(x - y)$$

Similarly, $uv\ \in f^{-1}(xy)$.

$$\mu_{f(A)_i}(x - y) = \begin{cases} \sup\limits_{z\epsilon f^{-1}(x - y)}\{\mu_{A_i}(z)\} & if\ f^{-1}(x - y) \neq \emptyset \\ 0 & otherwise \end{cases}$$

and

$$\vartheta_{f(A)_i}(x - y) = \begin{cases} \inf\limits_{z\epsilon f^{-1}(x - y)}\{\vartheta_{A_i}(z)\} & if\ f^{-1}(x - y) \neq \emptyset \\ 1 & otherwise \end{cases}$$

Consider $\mu_{f(A)_i}(x - y) = \sup\limits_{z\epsilon f^{-1}(x - y)} \mu_{A_i}(z)$

$$= \sup\limits_{u - v\epsilon f^{-1}(x - y)} \mu_{A_i}(u - v)$$

$$\geq \sup\limits_{u - v\epsilon f^{-1}(x - y)} (\mu_{A_i}(u) \wedge \mu_{A_i}(v))$$

$$\geq \sup\limits_{u\epsilon f^{-1}(x)} (\mu_{A_i}(u)) \wedge \sup\limits_{v\epsilon f^{-1}(y)} (\mu_{A_i}(v))$$

$$\geq \mu_{f(A)_i}(x) \wedge \mu_{f(A)_i}(y)$$

Hence $\mu_{f(A)_i}(x - y) \geq \mu_{f(A)_i}(x) \wedge \mu_{f(A)_i}(y)\ for\ all\ i$ and for all $x, y \in\ \mathbb{R}'$.

Similarly we can prove the conditions (ii), (iii) and (iv) of definition (13).

Hence $f(A)$ is a $k -$ intuitionistic fuzzy ideal of $\mathbb{R}'$.

Theorem:3.6 Let $\mathbb{R}$ and $\mathbb{R}'$ be two rings and $f: \mathbb{R} \to \mathbb{R}'$ be an onto homomorphism. Let A' be a $k -$ intuitionistic fuzzy ideal of $\mathbb{R}'$. Then $f^{-1}(A')$ is a $k -$ intuitionistic fuzzy ideals of $\mathbb{R}$.

Proof:

Let $A' = (\mu_{A_1}{}', \mu_{A_2}{}', \dots, \mu_{A_k}{}', \vartheta_{A_1}{}', \vartheta_{A_2}{}', \dots, \vartheta_{A_k}{}'$ be a $k -$ intuitionistic fuzzy ideal of $\mathbb{R}'$.

Then $f^{-1}(A') = (\mu_{f^{-1}(A)_1}{}', \mu_{f^{-1}(A)_2}{}', \dots, \mu_{f^{-1}(A)_k}{}', \vartheta_{f^{-1}(A)_1}{}', \vartheta_{f^{-1}(A)_2}{}', \dots, \vartheta_{f^{-1}(A)_k}{}'$ where $\mu_{f^{-1}(A)_i}{}'(x) = \mu_{A_i}{}'(f(x))$ and $\vartheta_{f^{-1}(A)_i}{}'(x) = \vartheta_{A_i}{}'(f(x))$

Let $x, y \in\ \mathbb{R}$

Now, $\mu_{f^{-1}(A)_i}{}'(x-y) = \mu_{A_i}{}'f(x-y)$

$$= \mu_{A_i}{}'\big(f(x)-f(y)\big)$$
$$\geq \mu_{A_i}{}'(f(x)) \wedge \mu_{A_i}{}'(f(y))$$
$$= \mu_{f^{-1}(A)_i}{}'(x) \wedge \mu_{f^{-1}(A)_i}{}'(y)$$

Hence $\mu_{f^{-1}(A)_i}{}'(x-y) \geq \mu_{f^{-1}(A)_i}{}'(x) \wedge \mu_{f^{-1}(A)_i}{}'(y)$

Now, $\mu_{f^{-1}(A)_i}{}'(xy) = \mu_{A_i}{}'f(xy)$

$$= \mu_{A_i}{}'\big(f(x).f(y)\big)$$
$$\geq \mu_{A_i}{}'(f(x)) \vee \mu_{A_i}{}'(f(y))$$
$$= \mu_{f^{-1}(A)_i}{}'(x) \vee \mu_{f^{-1}(A)_i}{}'(y)$$

Hence $\mu_{f^{-1}(A)_i}{}'(xy) \geq \mu_{f^{-1}(A)_i}{}'(x) \vee \mu_{f^{-1}(A)_i}{}'(y)$

Similarly we can prove $\vartheta_{f^{-1}(A)_i}{}'(x-y) \leq \vartheta_{f^{-1}(A)_i}{}'(x) \vee \vartheta_{f^{-1}(A)_i}{}'(y)$

$$\vartheta_{f^{-1}(A)_i}{}'(xy) \leq \vartheta_{f^{-1}(A)_i}{}'(x) \wedge \vartheta_{f^{-1}(A)_i}{}'(y)$$

Hence $f^{-1}(A')$ is a $k-$ intuitionistic fuzzy ideals of $\mathbb{R}$.

Theorem : 3.7

Let $\mathbb{R}$ be any ring and A be a fuzzy ideal of a ring $\mathbb{R}$ and if $\mu_{A_i}(x) < \mu_{A_i}(y)$ for some $x, y \in \mathbb{R}$. Then $\mu_{A_i}(x-y) = \mu_{A_i}(x) = \mu_{A_i}(y-x)\ for\ all\ i = 1,2,\dots,k$

Proof:

Let $A = (\mu_{A_1}, \mu_{A_2}, \dots \mu_{A_k}, \vartheta_{A_1}, \vartheta_{A_2}, \dots \vartheta_{A_k})$ be a $k-$ intuitionistic fuzzy ideal of $\mathbb{R}$.

Let $\mu_{A_i}(x) < \mu_{A_i}(y)$ for some $x, y \in \mathbb{R}$

Claim: $\mu_{A_i}(x-y) = \mu_{A_i}(x) = \mu_{A_i}(y-x)\ for\ all\ i = 1,2,\dots,k$

Now $\mu_{A_i}(x-y) \geq \mu_{A_i}(x) \wedge \mu_{A_i}(y)$

$$= \mu_{A_i}(x) \dots\dots\dots..(1)$$

And consider $\mu_{A_i}(x) = \mu_{A_i}(xyy^{-1})$

$$= \mu_{A_i}((xy)y^{-1})$$
$$\geq [\mu_{A_i}(xy) \wedge \mu_{A_i}(y^{-1})]$$
$$\geq [\mu_{A_i}(x) \wedge \mu_{A_i}(y)] \wedge \mu_{A_i}(y^{-1})$$
$$\geq \mu_{A_i}(x) \wedge \mu_{A_i}(y)]$$
$$= \mu_{A_i}(x-y) \dots\dots\dots..(2)$$

From (1) and (2) implies, we get, $\mu_{A_i}(x-y) = \mu_{A_i}(x) \dots\dots\dots(3)$

Similarly, we can prove, $\mu_{A_i}(x) = \mu_{A_i}(y-x) \dots\dots\dots.(4)$

From (3) and (4) implies, we get, $\mu_{A_i}(x-y) = \mu_{A_i}(x) = \mu_{A_i}(y-x)$

Definition: 3.8

Let $\mathbb{R}$ be a ring and A be a $k-$intuitionistic fuzzy ideal of $\mathbb{R}$. Let $t \in [0,1]$ and $t \leq \mu_{A_i}(e) + \vartheta_{A_i}(e)$. The $k-$intuitionistic fuzzy ideal A_t is called $k-$intuitionistic level ideal of A.

Theorem: 3.9

Let $\mathbb{R}$ be a ring. Let A_t and A_s are two $k-$intuitionistic level ideals of $\mathbb{R}$ are equal if and only if there is no x in $\mathbb{R}$ such that $s \leq \mu_{A_i}(x) + \vartheta_{A_i}(x) < t$.

Proof:
Let $A = (\mu_{A_1}, \mu_{A_2}, \dots \mu_{A_k}, \vartheta_{A_1}, \vartheta_{A_2}, \dots \vartheta_{A_k})$ be a $k-$ intuitionistic fuzzy ideal of $\mathbb{R}$.
Let A_t and A_s are two k −intuitionistic level ideals of $\mathbb{R}$.
Suppose $A_t = A_s$ $(s < t)$
Claim: There is no x in $\mathbb{R}$ such that $s \leq \mu_{A_i}(x) + \vartheta_{A_i}(x) < t$
Assume that x in $\mathbb{R}$ such that $s \leq \mu_{A_i}(x) + \vartheta_{A_i}(x) < t$

$\Longrightarrow \mu_{A_i}(x) + \vartheta_{A_i}(x) \geq s$ and $\mu_{A_i}(x) + \vartheta_{A_i}(x) < t$

$\Longrightarrow x \in A_s$ and $x \notin A_t$

$\Longrightarrow A_s \neq A_t$

Which is the contradiction to our assumption
Therefore there is no x in $\mathbb{R}$ such that $s \leq \mu_{A_i}(x) + \vartheta_{A_i}(x) < t$
Conversely,
Suppose that there is no x in $\mathbb{R}$ such that $s \leq \mu_{A_i}(x) + \vartheta_{A_i}(x) < t$
Claim: $A_t = A_s$
Suppose that $A_s \neq A_t$
If $s < t$ then $A_t \subset A_s$
For,
Let $x \in A_t$

$\Longrightarrow \mu_{A_i}(x) + \vartheta_{A_i}(x) \geq t > s$

$\Longrightarrow \mu_{A_i}(x) + \vartheta_{A_i}(x) > s$

Then $\Longrightarrow x \in A_s$

$\Longrightarrow A_t \subset A_s$

It is enough to prove that $A_t \not\subset A_s$

$\Longrightarrow x \in A_s$ and $x \notin A_t$

$\Longrightarrow \mu_{A_i}(x) + \vartheta_{A_i}(x) \geq s$ and $\mu_{A_i}(x) + \vartheta_{A_i}(x) < t$

Thus there exists an element in $\mathbb{R}$ such that $s \leq \mu_{A_i}(x) + \vartheta_{A_i}(x) < t$
Which is the contradiction to our assumption
Therefore, $A_t = A_s$.

4. CONCLUSION :

Necessity is the mother of invention. In this fast moving world, the necessity of the fuzzy theory became unavoidable. Every man in his day-to-day life, wants to find some thing new and different from others; the new structure developed here is one of such kind. In a short span of time, we made a study on the fuzzy theory and intuitionistic fuzzy theory. We define a new structure called a k −intuitionistic fuzzy subset and developed the respective theory mainly in the context of algebra. This can be further developed wherever fuzzy theory can be discussed.

REFERENCES :

[1] K. T. Atanassov, Intuitionistic Fuzzy sets, Fuzzy sets and systems, 20(1), (1986), 87-96.
[2] K. T. Atanassov, New operations defined over the Intuitionistic Fuzzy sets, Fuzzy sets and systems, 61, (1994), 137-142.
[3] I. N. Herstein, Topics in Algebra, Wiley-India, 1999.

[4] Rajesh Kumar, Fuzzy Subgroups, Fuzzy Ideals, and Fuzzy Cosets: Some Properties, Fuzzy Sets and Systems, 48(2), (1992), 267-274.
[5] P. Suseela, M. Shakthiganesan and R. Vembu, k- intuitionistic fuzzy structures, ISSN 2367-8283, Vol.22, 2016, No. 1,13-26.
[6] Wang-jin Liu, Fuzzy Invariant Subgroups and Fuzzy Ideals, Fuzzy sets and systems, 8(2), (1982), 133-139
[7] L.A. Zadeh, Fuzzy Sets, Information and Control, 8, (1965), 338-353.

PREDICTION OF ANNUAL RAINFALL IN COIMBATORE DISTRICT BY USING MULTILINEAR REGRESSION MODEL

S. Santha[1] and T. Subasini[2]

[1]Department of Mathematics, Government Arts and Science College, \
Nagercoil- 629004, Tamil Nadu, India.
Email: santhawilliam14@gmail.com
[2]Research Scholar, Rani Anna Government Arts and Science College for Women,
(Affiliated to Manonmaniam Sundaranar University, Tirunelveli)
Tirunelveli.,Tamil Nadu, India.
Email: subasinit@gmail.com

ABSTRACT: *Prediction of rainfall is the implementation of using science and technology to predict the state of the atmosphere. It is critical to calculating rainfall to effectively use water resources, agriculture production, and water structure development. The Regression technique makes several valuable contributions to the forecasting problem's solution. It also calculates the dependent variable's value estimates based on the independent variable's values. This article develops a multiple linear regression model for annual rainfall in Coimbatore District. The study represents the referred mathematical process and the prediction by using weather variables as input information. The model has developed using data from 2001 to 2020, and testing forecasts the rainfall intensity over the following years.*

Keywords: *Annual Rainfall, Multi Linear Regression, Weather variables.*

1. INTRODUCTION

Rainfall significantly affects the activities of human life. The diversity is quite large and characterizes the climate in the Coimbatore district in Tamil Nadu. Global climate change can increase the incidence of extreme rainfall. The analysis is needed to obtain rainfall prediction information that is very useful for reducing the impact of possible extreme rain events. Prediction of rainfall is still a considerable challenge to climatologists. However, it is the essential component of a climate system. Most of the burning issues of our time, like a global warning, floods, drought, heatwaves, soil erosion, and many other climatic problems, are directly related to rainfall. Agriculture is still the primary source of economic activities in most countries of the world, and rainfall increases crop production and protects the crops, human life, and the ecosystem. There is an increasing demand from policymakers for a reliable prediction of precipitation. Therefore it is vital to be able to predict rainfall correctly.

Time series data forecasting is a part of statistical modeling widely used in various fields because of its benefits in decision making. Time series analysis has several objectives, namely forecasting modeling and control. Forecasting is an element that is important in decision-making activities because whether or not an effective decision is made depends on several factors that influence it. However, unseen when a decision is taken. The purpose of time series forecasting is to predict the future values of certain variables that vary with time using their previous values. Forecasting is related to the formation of models and methods that can be used to produce a good forecast. The use of time-series data for forecasting is based on the behavior of past events.

In time-series data, the behavior of past events can be used for forecasting because it is expected that, in the future, the influence of the behavior of past events will still occur. The benefits of forecasting can be felt in many fields, including economics, finance, marketing, and production. Generally, time-series research uses linear time series models. Specifically, linear regression is a predictive statistical approach for modeling the relationship between a dependent variable with a given set of independent variables. Regression models are often used for estimating future events or values. Regression analysis includes parametric methods such as linear and logistic regression. Non-parametric methodologies such as projection pursuit, additive models, multivariate adaptive regression, etc., have also been applied to estimation and prediction problems (Holmstrom et al. 1997).

Regression analysis yields estimations of the dependent variable's values based on the independent variable's values. First, measure the strength of the regression relationship between y and the x variable. Then, the regression line depicts the average association between the variables X and Y. To determine which of the x variables are significant in predicting input into the equation; the equation produces estimations of the dependent variable. When values of the independent variable are entered into the equation, the equation makes measures of the dependent variable. The current study uses Multi Linear Regression model to forecast yearly rainfall in Coimbatore District from 2001 to 2020.

2. METHODOLOGY

Area of study and Data collection

Coimbatore district is in the western part of Tamil Nadu, enclosed by the Western Ghats mountain range on the west and north. The district's boundary is Palakkad in the west, Nilgiris in the north, erode district in the northeast, and south is Idukki. The rest of the section lies in the rain shadow region of the Western Ghats and experiences salubrious climate most parts of the year. The mean maximum and minimum temperatures for Coimbatore city during summer and winter vary between 35 °C to 18 °C. The average annual rainfall in the plains is around 700 mm. The data was collected from the statistical department of Coimbatore from 2001 to 2020.

For this current study weather parameters of the Coimbatore district were used which are Rainfall, Maximum temperature, Minimum temperature and wind speed.

Table 1: Data collection from 2001 to 2020

Year	Rainfall(y)	Wind Speed(x1)	Average Temperature(x2)
2001	752.8	18	26.96759259
2002	665.7	18.75	27.03703704
2003	644.7	29.58333333	27.06018519
2004	959.8	18.58333333	27.12962963
2005	973.5	19.41666667	25.18518519
2006	924.6	17.08333333	26.85185185

2007	863.1	14.16666667	26.71296296
2008	725.3	13.25	27.03703704
2009	798.1	14.25	26.96759259
2010	1165.8	14.41666667	26.94444444
2011	1177.8	15.5	26.99074074
2012	902.5	16.33333333	27.08333333
2013	806.6	17.58333333	27.08333333
2014	619.9	20.16666667	27.12962963
2015	1000.7	16.91666667	27.03703704
2016	1418.1	18.25	27.12962963
2017	791.5	18	27.15277778
2018	952	18.83333333	27.10648148
2019	859.5	18	27.03703704
2020	638	18.5	27.10647963

Multiple Regression

A multiple linear regressions analysis is working out to predict the values of a dependent variable, Y, given a set of k explanatory variables ($x_1, x_2 \ldots, x_k$).

$y = \theta_0 + \theta_1 X_1 + \theta_2 X_2 + \cdots\cdots\cdots\cdots + \theta_k X_k + \varepsilon$ ……………. (1)

Where y → dependent / response variables

X_i → independent / explanatory variables,

θ_i → determine the partial contribution of each of the x variable

ε → is the random error term

$$\theta_1 = \frac{\partial y}{\partial x1}, \theta_2 = \frac{\partial y}{\partial x2}, \ldots\ldots\ldots\ldots\ldots.. \theta_k = \frac{\partial y}{\partial xk}$$

Three variable model

$$y = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \varepsilon$$

where Y denotes a dependent variable, x_1 denote the first independent variable, x_2 denote the second independent variable

We can obtain the parameter estimates by normalizing the above regression equation.

$$\Sigma y = n\theta_0 + \theta_1 \Sigma x_1 + \theta_2 \Sigma x_2$$
$$\Sigma x_1 y = \theta_0 \Sigma x_1 + \theta_1 \Sigma x_1^2 + \theta_2 \Sigma x_1 x_2$$
$$\Sigma x_2 y = \theta_0 \Sigma x_2 + \theta_1 \Sigma x_1 x_2 + \theta_2 \Sigma x_2^2$$

In matrix form

$$\begin{bmatrix} n & \Sigma x_1 & \Sigma x_2 \\ \Sigma x_1 & \Sigma x_1^2 & \Sigma x_1 x_2 \\ \Sigma x_2 & \Sigma x_1 x_2 & \Sigma x_2^2 \end{bmatrix} \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} \Sigma y \\ \Sigma x_1 y \\ \Sigma x_2 y \end{bmatrix}$$

Using Cramer's rule

$$\theta_1 = \frac{\begin{bmatrix} n & \Sigma y & \Sigma x_2 \\ \Sigma x_1 & \Sigma x_1 y & \Sigma x_1 x_2 \\ \Sigma x_2 & \Sigma x_2 y & \Sigma x_2^2 \end{bmatrix}}{\begin{bmatrix} n & \Sigma x_1 & \Sigma x_2 \\ \Sigma x_1 & \Sigma x_1^2 & \Sigma x_1 x_2 \\ \Sigma x_2 & \Sigma x_1 x_2 & \Sigma x_2^2 \end{bmatrix}} \qquad \theta_2 = \frac{\begin{bmatrix} n & \Sigma x_1 & \Sigma y \\ \Sigma x_1 & \Sigma x_1^2 & \Sigma x_1 y \\ \Sigma x_2 & \Sigma x_1 x_2 & \Sigma x_2 y \end{bmatrix}}{\begin{bmatrix} n & \Sigma x_1 & \Sigma x_2 \\ \Sigma x_1 & \Sigma x_1^2 & \Sigma x_1 x_2 \\ \Sigma x_2 & \Sigma x_1 x_2 & \Sigma x_2^2 \end{bmatrix}}$$

Multiple Linear Regression in Linear Algebra Notation

$$y = x\theta + \varepsilon$$

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix}$$

The response value for all observations is n×1 dimensional vectors.

$$x = \begin{bmatrix} 1 & x_{11} & x_{12} & \cdot & \cdot & x_{1k} \\ 1 & x_{21} & x_{22} & \cdot & \cdot & x_{2k} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & x_{n1} & x_{n2} & \cdot & \cdot & x_{nk} \end{bmatrix}$$

The intercept and slopes are k×1 dimensional vectors denoted by 'θ'

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \cdot \\ \cdot \\ \theta_k \end{bmatrix}$$

All the error term has an n×1 dimensional vector denoted by ε

$$\varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \cdot \\ \cdot \\ \varepsilon_n \end{bmatrix}$$

Here we use the method of Ordinary Least Square

MLR is $y = \theta_0 + \theta_1 X_1 + \theta_2 X_2 + \cdots\cdots\cdots\cdots + \theta_k X_k + \varepsilon$

Using the OLS method, the objective is to obtain estimates $(\theta_1, \theta_2, \theta_3 \ldots\ldots\ldots.\theta_k)$ by Minimizing $SSE = \Sigma e^2 = \Sigma(y_i - \text{predicted } y)^2$

The parameter estimates are said to be the best linear unbiased estimates can then be used in the prediction equation.

3. Result and Discussion :

To establish the multilinear equation using MS Excel under the dependent variable rainfall corresponding to independent variables Temperature and wind speed data from 2001 to 2020.

	Coefficients	Standard Error	t Stat	P-value
Intercept	2792.568685	2957.269022	0.94430661	0.358242
Wind speed	-18.6588644	13.75970183	-1.356051506	0.192823
Average Temperature	-58.61084474	109.2603048	-0.536433107	0.59861

In the rainfall factors, we have used by multiple regression approach. This approach will select rainfall data and other climate factors in the Coimbatore district. Applying a multiple linear regression method to the data set and finding an approximate equation between rainfall and climate variables. So the Estimated MLR is

$\text{Rainfall} = 2792.57 + (-18.6589 * x_1) + (-58.611 * x_2)$ ………..(2)

From equation (2), we can predict the rainfall for future years by using wind speed and temperature.

Table: 2 comparisons between Actual and Predict value

Observation	Actual Rainfall	Predicted Rainfall	Percentage of Error
1	752.8	876.1157431	16.31
2	665.7	858.0513972	28.89
3	644.7	654.5569671	1.53
4	959.8	955.7342779	0.42
5	973.5	954.1507557	1.99
6	924.6	900.0033647	2.66
7	863.1	962.5654476	11.52
8	725.3	860.6751514	18.66
9	798.1	746.0864846	6.52
10	1165.8	1044.3334064	10.42
11	1177.8	1021.4061716	13.27
12	902.5	900.4301878	0.23
13	806.6	877.1066073	8.74
14	619.9	726.1910759	17.15
15	1000.7	992.2593153	0.843
16	1418.1	1452.9538994	2.46
17	791.5	800.2618829	1.107
18	952	952.4262943	0.045
19	859.5	872.0455455	1.45
20	638	658.6460243	3.23

From the above table the percentage of Error is 7.38

Table: 3 Results of Multi linear regression

Regression Statistics	
R Square(co efficient of Determination)	0.819648
Adjusted R Square	0.7984
Observations	20

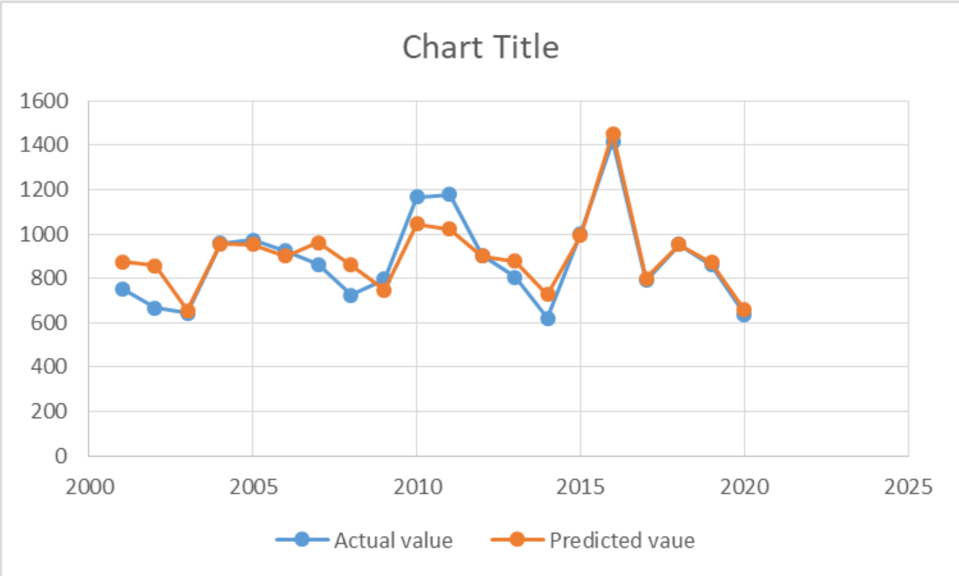


Figure 1: Comparison between actual and predict value

4. CONCLUSION:

Regression analysis is a quantitative analysis of the relationship between response and explanatory variables. Multi Linear Regression is an extension of simple Regression. The analysis aims to determine the connection between rainfall and weather variables such as max temperature, min temperature, and wind speed. That is, to examine the functional relationship between the variables and, as a result, to develop a prediction mechanism. This research uses a regression model to explain occurrence analysis with an accuracy of 92.62 percent, which may utilize the model to create yearly precipitation projections. In addition, the precipitation model used provided information regarding the Coimbatore District's water resources and agriculture.

REFERENCES:

[1] N. Sen, "Forecast models for Indian South-West Monsoons Season Rainfall", in Current Science, vol. 84, No. 10, May 2003, pp.1290-1291.

[2] S. Nkrintra, et al., "Seasonal Forecasting of Thailand Summer Monsoon Rainfall", in International Journal of Climatology, Vol. 25, Issue 5,American Meteorological Society, 2005, pp. 649-664.

[3] T. Sohn, J. H. Lee, S. H. Lee, C. S. Ryu, "A statistical Prediction of Heavy Rain in South Korea", in Advances in Atmospheric Sciences, Vol. 22, No. 5, 2005, pp.703-710

[4] M. T. Mebrhatu, M. Tsubo, S. Walker, "A Statistical Model for Rainfall Prediction over the Highlands of Eritrea", in International Crop Science Organization.
[5] Mrs. C. Beulah Christalin Latha "A Service Oriented Architecture for Weather Forecasting Using Data Mining" Int. J. of Advanced Networking and Applications Vol: 02, Issue:02, Pages:608-613 (2010)
[6] Dr. S. Santha and T.Subasini "Prediction of Annual Rainfall in Erode District using Regression Model" Proceedings of the International Conference on Advances and Applications in Mathematical Sciences.
[7] MAI Navix, NH Niloy." Multiple linear Regressions for Predicting Rainfall for Bangladesh".Communications, Vol.6 No.1, 2018, pp.1-4.
[8] Enock Mintah Ampaw "Time Series Modelling of Rainfall in New Juaben Municipality of the Eastern Region of Ghana"in Current Science, vol. 84, No. 10, May 2003, pp.1290-1291.
[9] Ajith Abraham, Dan Steinberg and Ninan Sajeeth Philip "Rainfall Prediction Using Software Computing Model and Multiple Adaptive Regression Splines" International Journal of Engineering and Technology , 2010, pp.510-534.
[10] Folorunsho Olaiya" Application of Data Mining Techniques in Weather Prediction and Climate Change Studies "I.J. Information Engineering and Electronic Business, 2012, 1, 51-59.
[11] Paras et.al, (2012) "A simple Weather Forecasting Model Using Mathematical Regression'' In Bangladesh Research Journal of Extension Education Special Issue (Volume 1). January 2012.
[12] H.Hasani, et.al, (2008) A New Approach to polynomial Regression and its Application to Physical growth of Human Height.
[13] Kazi Md. Jahid Hasan "Rainfall Forecasting of monsoon season of Sylhet city using Regression", International Research Journal of Modernization in Engineering Technology and science, Vol.3/issue.3/march 2020.

PERTURBATION TECHNIQUES FOR THE TRANSMISSION DYNAMICS OF ZIKA VIRUS MATHEMATICAL MODEL

K. Vaishnavi[1] and R. Malini Devi[2]

[1,,2]Department of Mathematics,
The Standard Fireworks Rajaratnam College For Women,
Sivakasi - 626 123, Tamilnadu, India.
[1]First author: Email id: 18vaishnavik-pgmat@sfrcollege.edu.in
[2]Corresponding author: Email id: malinidevi-mat@sfrcollege.edu.in

ABSTRACT: *In this paper, a mathematical model representing the transmission dynamics of Zika virus between the human (S_h, E_h, I_h, R_h) and the vector (mosquito) (S_v, E_v, I_v) population are considered. The main objective of this paper is to implement the perturbation techniques such as Homotopy Perturbation Method (HPM) and New Homotopy Perturbation Method (NHPM) to obtain the analytical solution of the model. Each of the parameters involved in the transmission of the Zika virus are analyzed with the help of the perturbation technique. As the recovery rate of the infected human population increases, the human population becomes free from the transmission of the Zika virus.*

Keywords: *Transmission dynamics, Human and vector population, Zika virus, Aedes mosquito, Recovery rate, Perturbation techniques.*

1. INTRODUCTION:

The Flaviviridae family and Flavivirus species include the Zika virus. Aedes mosquitoes that are active during the day, like A. aegypti and A. albopictus, transmit it. The Zika Forest in Uganda, where the virus was first discovered in 1947, gave the disease its name. Zika was first discovered in Uganda in 1947 in primates; it was then discovered in people in 1952. Dengue, yellow fever, Japanese encephalitis, and West Nile viruses all belong to the same family as the Zika virus. It has been documented to happen within a constrained equatorial band stretching from Africa to Asia since the 1950s. The Zika virus pandemic of 2015–2016 was caused by the virus' eastward spread across the Pacific Ocean to the Americas between 2007 and 2016. The genome of the zika virus is non-segmented, single-stranded, and 10 kilobases in size. It is also enclosed and icosahedral and has positive-sense RNA genome [1-6].

Aedes mosquito species such as A. africanus, A. apicoargenteus, A. furcifer, A. hensilli, A. luteocephalus, and A. vittatus are additionally transmitting the virus, which has an extrinsic incubation time of approximately 10 days in mosquitoes. With only infrequent transmission to people, the virus's host were monkeys and mosquitoes whose cycle is known as the enzootic cycle. Aedes aegypti mosquitoes are the main vectors of Zika, but it can also be shared through blood transfusions and sexual contact. Many Zika virus patients will not show any signs or only show minor ones. Fever, rash, headache, joint pain, red eyes, and muscle pain are among the most typical Zika signs. Days to a week can pass between the onsets of symptoms. Diagnosis of Zika is based on a person's recent travel history, symptoms, and test results. A blood or urine test can confirm a Zika infection. Symptoms of Zika are similar to other illnesses spread through mosquito bites, like dengue and chikungunya. Doctors or other healthcare providers may order tests to look for several types of infections [7-12].

The development of inactivated vaccines and other non-live vaccines that are safe to use in pregnant women has been recommended as a top priority by the World Health Organization. A vaccine against Zika was being developed by 18 companies and institutions as of March 2016, but they estimate it won't be generally accessible for another 10 years. The FDA first approved a human clinical study for a Zika vaccine in June 2016. A DNA vaccine received approval for phase-2 clinical studies in March 2017. This vaccine is made up of a plasmid, a microscopic circular fragment of DNA that expresses the genes for the Zika virus envelope proteins [13-24]. Section 2 provides the mathematical model of the Zika virus as proposed by S.K.Biswas et.al and all the parameters involved in it. Section 3 deals with the perturbation techniques like Homotopy Perturbation and New Homotopy Perturbation methods to derive the analytical solution of the model. In section 4, the analytical solution is verified for its accuracy with the numerical solution using the Matlab software and is represented in the form of graphs which clearly shows the response of the two population, human and vector concerning time under the variation of values in each parameter of the model.

2. TRANSMISSION DYNAMICS OF ZIKA VIRUS MATHEMATICAL MODEL

Let the total human population $N_h(t)$ is classified into four compartments comprised of susceptible human $S_h(t)$, exposed human $E_h(t)$, infected human $I_h(t)$ and recovered human $R_h(t)$. Here consider a human individual who recovered from the infection of the Zika virus gain lifelong immunity from it. Since only female mosquitoes spreads the Zika infection so the total female mosquito population $N_v(t)$ is divided into three compartments viz. susceptible mosquitoes $S_v(t)$, exposed mosquitoes $E_v(t)$ and infected mosquitoes $I_v(t)$. Again recovery of mosquitoes from Zika infection is not taken into consideration due to its short life span.

Let π be the constant recruitment rate of susceptible humans and μ is the natural death rate of the human population. Suppose, susceptible individuals acquire infection due to effective contact with an infected vector at the rate $\lambda_1 = \frac{b_2 \alpha_1 I_v}{N_h}, \lambda_2 = \frac{c \alpha_2 I_v}{N_h}$ be the infection due to sexual interaction with the infected individuals and susceptible humans become aware at a constant, a & enter into recovered class R_h. So the total infection strength of humans is $\lambda_h=\lambda_1+\lambda_2$. Here assume that the susceptible mosquitoes acquire infection at a rate $\lambda_v S_v$ from infected humans where $\lambda_v = \frac{b_2 \alpha_3 I_h}{N_h}$.

The transmission dynamics of the Zika virus between the human and the vector population [25] can be represented by the following system of non-linear differential equations:

$$\frac{dS_h}{dt} = \pi - (\lambda_1 + \lambda_2) S_h - (\mu + a) S_h \tag{2.1}$$

$$\frac{dE_h}{dt} = (\lambda_1 + \lambda_2) S_h - (\sigma + \mu) E_h \tag{2.2}$$

$$\frac{dI_h}{dt} = \sigma E_h - (\gamma + \mu) I_h \tag{2.3}$$

$$\frac{dR_h}{dt} = \gamma I_h - \mu R_h + aS_h \tag{2.4}$$

$$\frac{dS_v}{dt} = \pi_1 - \lambda_v S_v - (\mu_1 + b)S_v \tag{2.5}$$

$$\frac{dE_v}{dt} = \lambda_v S_v - (\sigma_1 + \mu_1 + b)E_v \tag{2.6}$$

$$\frac{dI_v}{dt} = \sigma_1 E_v - (\mu_1 + b)I_v \tag{2.7}$$

The initial condition at the time, $t=0$, $S_h = S_{h0}, E = E_{h0}, I_h = I_{h0}, R_h = R_{h0}, S_v = S_{v0}$, $I_v = I_{v0}$, $E_v = E_{v0}$.

(2.8)

Parameters	Description
N_h	Total human population
S_h	Susceptible human population
E_h	Exposed human population
I_h	Infected human population
R_h	Recovered human population
N_v	Total vector population
S_v	Susceptible vector population
E_v	Exposed vector population
I_v	Infected vector population
π,π1	The recruitment rate of humans and mosquitoes respectively
μ,μ1	The natural death rate of humans and mosquitoes respectively
α1	Transmission probability per biting of Susceptible humans with infected mosquito
α3	Transmission probability per biting of Susceptible mosquito with infected humans.
c	Sexual contact rate between a susceptible human to an infected human
α2	Transmission probability per sexual contact- among a susceptible and infected human
σ	Progression rate from exposed to infected human
γ	The recovery rate of infected human
a	Rate of awareness in the host population
σ1	Progression rate from exposed to infected mosquito
b	Constant rate of effective mosquito control

Table 1 List of Parameters

3 PERTURBATION TECHNIQUES FOR OBTAINING ANALYTICAL SOLUTION

Perturbation techniques like Homotopy Perturbation Method and New Homotopy Perturbation Method [26-31] are used to derive the analytical solution of the Zika virus mathematical model equation from (2.1) to (2.7).

3.1 HOMOTOPY PERTURBATION METHOD

To find the solution of equation (2.1) - (2.7) construct the homotopy as follows:

$$(1-p)\left[\frac{dS_h}{dt}+(\lambda_1+\lambda_2)S_h+(\mu+a)S_h\right]+p\left[\frac{dS_h}{dt}-\pi+(\lambda_1+\lambda_2)S_h+(\mu+a)S_h\right]=0 \tag{3.1}$$

$$(1-p)\left[\frac{dE_h}{dt}+(\sigma+\mu)E_h\right]+p\left[\frac{dE_h}{dt}-(\lambda_1+\lambda_2)S_h+(\sigma+\mu)E_h\right] \tag{3.2}$$

$$(1-p)\left[\frac{dI_h}{dt}+(\gamma+\mu)I_h\right]+p\left[\frac{dI_h}{dt}-\sigma E_h+(\gamma+\mu)I_h\right]=0 \tag{3.3}$$

$$(1-p)\left[\frac{dR_h}{dt}+\mu R_h\right]+p\left[\frac{dR_h}{dt}+\mu R_h-\gamma I_h-aS_h\right]=0 \tag{3.4}$$

$$(1-p)\left[\frac{dS_v}{dt}+\lambda_v S_v+(\mu_1+b)S_v\right]+p\left[\frac{dS_v}{dt}-\pi_1+\lambda_v S_v+(\mu_1+b)S_v\right]=0 \tag{3.5}$$

$$(1-p)\left[\frac{dE_v}{dt}+(\sigma_1+\mu_1+b)S_v\right]+p\left[\frac{dE_v}{dt}-\lambda_v S_v+(\sigma_1+\mu_1+b)S_v\right]=0 \tag{3.6}$$

$$(1-p)\left[\frac{dI_v}{dt}+(\mu_1+b)I_v\right]+p\left[\frac{dI_v}{dt}-\sigma_1 E_v+(\mu_1+b)I_v\right]=0 \tag{3.7}$$

The solution of equations (2.1) - (2.7) is written as a power series as follows:

$$S_h=S_{h0}+pS_{h1}+\ldots \tag{3.8}$$

$$E_h=E_{h0}+pE_{h1}+\ldots \tag{3.9}$$

$$I_h=I_{h0}+pI_{h1}+\ldots \tag{3.10}$$

$$R_h=R_{h0}+pR_{h1}+\ldots \tag{3.11}$$

$$S_v=S_{v0}+pS_{v1}+.. \tag{3.12}$$

$$E_v=E_{v0}+pE_{v1}+\ldots \tag{3.13}$$

$$I_v=I_{v0}+pI_{v1}+\ldots \tag{3.14}$$

Substituting the equations (3.8)-(3.14) in (3.1)-(37) we get,

$$(1-p)\left[\frac{d(S_{h0}+pS_{h1}+\ldots)}{dt}+(\lambda_1+\lambda_2)(S_{h0}+pS_{h1}+\ldots)+(\mu+a)(S_{h0}+pS_{h1}+\ldots)\right]$$
$$+p\left[\frac{dS_h}{dt}-\pi+(\lambda_1+\lambda_2)(S_{h0}+pS_{h1}+\ldots)+(\mu+a)(S_{h0}+pS_{h1}+\ldots)\right]=0 \tag{3.15}$$

$$(1-p)\left[\frac{d(E_{h0}+pE_{h1}+...)}{dt}+(\sigma+\mu)(E_{h0}+pE_{h1}+...)\right]$$

$$+p\left[\frac{dE_h}{dt}-(\lambda_1+\lambda_2)(S_{h0}+pS_{h1}+...)+(\sigma+\mu)(E_{h0}+pE_{h1}+...)\right]=0 \quad (3.16)$$

$$(1-p)\left[\frac{d(I_{h0}+pI_{h1}+...)}{dt}+(\gamma+\mu)(I_{h0}+pI_{h1}+...)\right]$$

$$+p\left[\frac{d(I_{h0}+pI_{h1}+...)}{dt}-\sigma(E_{h0}+pE_{h1}+...)+(\gamma+\mu)(I_{h0}+pI_{h1}+...)\right]=0 \quad (3.17)$$

$$(1-p)\left[\frac{d(R_{h0}+pR_{h1}+...)}{dt}+\mu(R_{h0}+pR_{h1}+...)\right]$$

$$+p\left[\frac{d(R_{h0}+pR_{h1}+...)}{dt}+\mu(R_{h0}+pR_{h1}+...)-\gamma(I_{h0}+pI_{h1}+...)-a(S_{h0}+pS_{h1}+...)\right]=0$$

(3.18)

$$(1-p)\left[\frac{d(S_{v0}+pS_{v1}+...)}{dt}+\lambda_v(S_{v0}+pS_{v1}+...)+(\mu_1+b)(S_{v0}+pS_{v1}+...)\right]$$

$$+p\left[\frac{d(S_{v0}+pS_{v1}+...)}{dt}-\pi_1+\lambda_v(S_{v0}+pS_{v1}+...)+(\mu_1+b)(S_{v0}+pS_{v1}+...)\right]=0 \quad (3.19)$$

$$(1-p)\left[\frac{d(E_{v0}+pE_{v1}+...)}{dt}+(\sigma_1+\mu_1+b)(E_{v0}+pE_{v1}+...)\right]$$

$$+p\left[\frac{d(E_{v0}+pE_{v1}+...)}{dt}-\lambda_v(S_{v0}+pS_{v1}+...)+(\sigma_1+\mu_1+b)(E_{v0}+pE_{v1}+...)\right]=0 \quad (3.20)$$

$$(1-p)\left[\frac{d(I_{v0}+pI_{v1}+...)}{dt}+(\mu_1+b)(I_{v0}+pI_{v1}+...)\right]$$

$$+p\left[\frac{d(I_{v0}+pI_{v1}+...)}{dt}-\sigma_1(E_{v0}+pE_{v1}+...)+(\mu_1+b)(I_{v0}+pI_{v1}+...)\right]=0$$

(3.21)

Comparing the coefficients of p^0 of equations (3.15) - (3.21),

$$p^0 \ :\left[\frac{dS_{h0}}{dt}+(\lambda_1+\lambda_2)S_{h0}+(\mu+a)S_{h0}\right]=0 \quad (3.22)$$

$$p^0 \ :\left[\frac{dE_{h0}}{dt}+(\sigma+\mu)E_{h0}\right]=0 \quad (3.23)$$

$$p^0 \ :\left[\frac{dI_{h0}}{dt}+(\gamma+\mu)I_{h0}\right]=0 \quad (3.24)$$

$$p^0 \ :\left[\frac{dR_{h0}}{dt}+\mu R_{h0}\right]=0 \quad (3.25)$$

$$p^0 \ :\left[\frac{dS_{v0}}{dt}+\lambda_v S_{v0}+(\mu_1+b)S_{v0}\right]=0$$

(3.26)

$$p^0 \ : \left[\frac{dE_{v0}}{dt} + (\sigma_1 + \mu_1 + b)E_{v0} \right] = 0 \tag{3.27}$$

$$p^0 \ : \left[\frac{d\,I_{v0}}{dt} + (\mu_1 + b)I_{v0} \right] = 0 \tag{3.28}$$

Using the initial condition, the solution of the equations (2.1) - (2.7) is given as follows:

$$S_{h0} = S_{hi}\, e^{-(\lambda_1 + \lambda_2 + \mu + a)t} \tag{3.29}$$

$$E_{h0} = E_{hi}\, e^{-(\sigma + \mu)t} \tag{3.30}$$

$$I_{h0} = I_{hi}\, e^{-(\gamma + \mu)t} \tag{3.31}$$

$$R_{h0} = R_{hi}\, e^{-\mu t} \tag{3.32}$$

$$S_{v0} = S_{vi}\, e^{-(\lambda_v + \mu_1 + b)t} \tag{3.33}$$

$$E_{v0} = E_{vi}\, e^{-(\sigma_1 + \mu_1 + b)t} \tag{3.34}$$

$$I_v = I_{vi} e^{(-(\mu_1 + b)t)} \tag{3.35}$$

$\therefore$ The Solution of Susceptible human (S_h), Exposed human (E_h), Infected human (I_h), Recovered human (R_h), Susceptible vector (S_v), Exposed vector (E_v) and Infected vector (I_v) is

$$S_h = S_{hi} e^{(-(\lambda_1+\lambda_2+\mu+a)t)} + \frac{\pi}{\lambda_1 + \lambda_2 + \mu + a} - \frac{\pi}{\lambda_1 + \lambda_2 + \mu + a} e^{(-(\lambda_1+\lambda_2+\mu+a)t)} \tag{3.36}$$

$$E_h = E_{hi} e^{(-(\sigma+\mu)t)} - \left(\frac{(\lambda_1 + \lambda_2)S_{hi} e^{(-(\lambda_1+\lambda_2+a+\mu)t)}}{\lambda_1 + \lambda_2 + a - \sigma} \right) + \left(\frac{(\lambda_1 + \lambda_2)S_{hi} e^{(-(\sigma+\mu)t)}}{\lambda_1 + \lambda_2 + a - \sigma} \right) \tag{3.37}$$

$$I_h = I_{hi} e^{(-(\gamma+\mu)t)} - \left(\frac{\sigma E_{h0} e^{(-(\sigma+\mu)t)}}{\sigma - \gamma} \right) + \left(\frac{\sigma E_{h0} e^{(-(\gamma+\mu)t)}}{\sigma - \gamma} \right) \tag{3.38}$$

$$R_h = R_{hi} e^{(-(\mu t)} - \left(\frac{\gamma I_{h0} e^{(-(\gamma+\mu)t)}}{\gamma} \right) - \left(\frac{a S_{h0} e^{(-(\lambda_1+\lambda_2+\mu+a)t)}}{\lambda_1 + \lambda_2 + a} \right) + \left(\left(\frac{\gamma I_{h0}}{\gamma} \right) + \left(\frac{a S_{h0}}{\lambda_1 + \lambda_2 + a} \right) \right) e^{(-(\mu t)} \tag{3.39}$$

$$S_v = S_{vi} e^{(-(\lambda_v+\mu_1+b)t)} + \left(\frac{\pi_1}{\lambda_v + \mu_1 + b} \right) - \left(\frac{\pi_1}{\lambda_v + \mu_1 + b} \right) e^{(-(\lambda_v+\mu_1+b)t)} \tag{3.40}$$

$$E_v = E_{vi} e^{(-(\sigma_1+\mu_1+b)t)} - \left(\frac{\lambda_v S_{vi} e^{(-(\lambda_v+\mu_1+b)t)}}{\lambda_v - \sigma_1} \right) + \left(\frac{\lambda_v S_{vi}}{\lambda_v - \sigma_1} \right) e^{(-(\sigma_1+\mu_1+b)t)} \tag{3.41}$$

$$I_v = I_{vi} e^{(-(\mu_1+b)t)} - \left(\frac{\sigma_1 E_{v0} e^{(-(\sigma_1+\mu_1+b)t)}}{\sigma_1} \right) + \left(\frac{\sigma_1 E_{v0}}{\sigma_1} \right) e^{(-(\mu_1+b)t)} \tag{3.42}$$

3.2 NEW HOMOTOPY PERTURBATION METHOD

To find the solution of equation (2.1) - (2.7) by the new homotopy as follows:

$$(1-p)\left[\frac{dS_h}{dt}-\pi+(\lambda_1+\lambda_2)S_h+(\mu+a)S_h\right]+p\left[\frac{dS_h}{dt}-\pi+(\lambda_1+\lambda_2)S_h+(\mu+a)S_h\right]=0 \tag{3.43}$$

$$(1-p)\left[\frac{dE_h}{dt}-(\lambda_1+\lambda_2)S_h(t=0)+(\sigma+\mu)E_h\right]+p\left[\frac{dE_h}{dt}-(\lambda_1+\lambda_2)S_h+(\sigma+\mu)E_h\right]=0$$

(3.44)

$$(1-p)\left[\frac{dI_h}{dt}-\sigma E_h(t=0)+(\gamma+\mu)I_h\right]+p\left[\frac{dI_h}{dt}-\sigma E_h+(\gamma+\mu)I_h\right]=0 \tag{3.45}$$

$$(1-p)\left[\frac{dR_h}{dt}+\mu R_h-\gamma I_h(t=0)-aS_h(t=0)\right]+p\left[\frac{dR_h}{dt}+\mu R_h-\gamma I_h-aS_h\right]=0 \tag{3.46}$$

$$(1-p)\left[\frac{dS_v}{dt}+\lambda_v S_v+(\mu_1+b)S_v\right]+p\left[\frac{dS_v}{dt}-\pi_1+\lambda_v S_v+(\mu_1+b)S_v\right]=0 \tag{3.47}$$

$$(1-p)\left[\frac{dE_v}{dt}-\lambda_v S_v(t=0)+(\sigma_1+\mu_1+b)E_v\right]+p\left[\frac{dE_v}{dt}-\lambda_v S_v+(\sigma_1+\mu_1+b)E_v\right]=0 \tag{3.48}$$

$$(1-p)\left[\frac{dI_v}{dt}-\sigma_1 E_v(t=0)+(\mu_1+b)I_v\right]+p\left[\frac{dI_v}{dt}-\sigma_1 E_v+(\mu_1+b)I_v\right]=0 \tag{3.49}$$

The solution of equations (2.1) - (2.7) is written as a power series as follows:

$$S_h=S_{h0}+pS_{h1}+... \tag{3.50}$$

$$E_h=E_{h0}+pE_{h1}+... \tag{3.51}$$

$$I_h=I_{h0}+pI_{h1}+... \tag{3.52}$$

$$R_h=R_{h0}+pR_{h1}+... \tag{3.53}$$

$$S_v=S_{v0}+pS_{v1}+.. \tag{3.54}$$

$$E_v=E_{v0}+pE_{v1}+... \tag{3.55}$$

$$I_v=I_{v0}+pI_{v1}+... \tag{3.56}$$

Substituting the equations (3.50)-(3.56) in (3.43)-(3.49) we get,

$$(1-p)\left[\frac{d(S_{h0}+pS_{h1}+...)}{dt}+(\lambda_1+\lambda_2)(S_{h0}+pS_{h1}+...)+(\mu+a)(S_{h0}+pS_{h1}+...)\right]$$
$$+p\left[\frac{dS_h}{dt}-\pi+(\lambda_1+\lambda_2)(S_{h0}+pS_{h1}+...)+(\mu+a)(S_{h0}+pS_{h1}+...)\right]=0 \tag{3.57}$$

$$(1-p)\left[\frac{d(E_{h0}+pE_{h1}+...)}{dt}-(\lambda_1+\lambda_2)S_h(t=0)+(\sigma+\mu)(E_{h0}+pE_{h1}+...)\right]$$
$$+p\left[\frac{dE_h}{dt}-(\lambda_1+\lambda_2)(S_{h0}+pS_{h1}+...)+(\sigma+\mu)(E_{h0}+pE_{h1}+...)\right]=0 \tag{3.58}$$

$$(1-p)\left[\frac{d(I_{h0}+pI_{h1}+...)}{dt}-\sigma E_h(t=0)+(\gamma+\mu)(I_{h0}+pI_{h1}+...)\right]$$

$$+p\left[\frac{d(I_{h0}+pI_{h1}+...)}{dt}-\sigma(E_{h0}+pE_{h1}+...)+(\gamma+\mu)(I_{h0}+pI_{h1}+...)\right]=0 \qquad (3.59)$$

$$(1-p)\left[\frac{d(R_{h0}+pR_{h1}+...)}{dt}+\mu(R_{h0}+pR_{h1}+...)-\gamma I_h(t=0)-aS_h(t=0)\right]$$

$$+p\left[\frac{d(R_{h0}+pR_{h1}+...)}{dt}+\mu(R_{h0}+pR_{h1}+...)-\gamma(I_{h0}+pI_{h1}+...)-a(S_{h0}+pS_{h1}+...)\right]=0$$

(3.60)

$$(1-p)\left[\frac{d(S_{v0}+pS_{v1}+...)}{dt}+\lambda_v(S_{v0}+pS_{v1}+...)+(\mu_1+b)(S_{v0}+pS_{v1}+...)\right]$$

$$+p\left[\frac{d(S_{v0}+pS_{v1}+...)}{dt}-\pi_1+\lambda_v(S_{v0}+pS_{v1}+...)+(\mu_1+b)(S_{v0}+pS_{v1}+...)\right]=0 \qquad (3.61)$$

$$(1-p)\left[\frac{d(E_{v0}+pE_{v1}+...)}{dt}-\lambda_v S_v(t=0)+(\sigma_1+\mu_1+b)(E_{v0}+pE_{v1}+...)\right]$$

$$+p\left[\frac{d(E_{v0}+pE_{v1}+...)}{dt}-\lambda_v(S_{v0}+pS_{v1}+...)+(\sigma_1+\mu_1+b)(E_{v0}+pE_{v1}+...)\right]=0 \qquad (3.62)$$

$$(1-p)\left[\frac{d(I_{v0}+pI_{v1}+...)}{dt}-\sigma_1 E_v(t=0)+(\mu_1+b)(I_{v0}+pI_{v1}+...)\right]$$

$$+p\left[\frac{d(I_{v0}+pI_{v1}+...)}{dt}-\sigma_1(E_{v0}+pE_{v1}+...)+(\mu_1+b)(I_{v0}+pI_{v1}+...)\right]=0$$

(3.63)

Comparing the coefficients of p^0 from equations (3.57) - (3.63),

$$p^0 \ :\left[\frac{dS_{h0}}{dt}+(\lambda_1+\lambda_2)S_{h0}+(\mu+a)S_{h0}\right]=\pi \qquad (3.64)$$

$$p^0 \ :\left[\frac{dE_{h0}}{dt}+(\sigma+\mu)E_{h0}\right]=(\lambda_1+\lambda_2)S_{h0} \qquad (3.65)$$

$$p^0 \ :\left[\frac{dI_{h0}}{dt}+(\gamma+\mu)I_{h0}\right]=\sigma E_{h0} \qquad (3.66)$$

$$p^0 \ :\left[\frac{dR_{h0}}{dt}+\mu R_{h0}\right]=\gamma I_h+aS_{h0} \qquad (3.67)$$

$$p^0 \ :\left[\frac{dS_{v0}}{dt}+\lambda_v S_{v0}+(\mu_1+b)S_{v0}\right]=\pi_1$$

(3.68)

$$p^0 \ :\left[\frac{dE_{v0}}{dt}+(\sigma_1+\mu_1+b)E_{v0}\right]=\lambda_v S_{v0}$$

(3.69)

$$p^0 \ :\left[\frac{dI_{v0}}{dt}+(\mu_1+b)I_{v0}\right]=\sigma_1 E_{v0}$$

(3.70)

$\therefore$ The Solution of Susceptible human (S_h), Exposed human (E_h), Infected human (I_h), Recovered human (R_h), Susceptible vector (S_v), Exposed vector (E_v) and Infected vector (I_v) is

$$S_h = \frac{\pi}{\lambda_1 + \lambda_2 + \mu + a} + \left(S_{h0} - \frac{\pi}{\lambda_1 + \lambda_2 + \mu + a} \right) e^{-(\lambda_1 + \lambda_2 + \mu + a)t} \tag{3.71}$$

$$E_h = \frac{(\lambda_1 + \lambda_2) S_{h0}}{\sigma + \mu} + \left(E_{h0} - \frac{(\lambda_1 + \lambda_2) S_{h0}}{\sigma + \mu} \right) e^{-(\sigma + \mu)t} \tag{3.72}$$

$$I_h = \frac{\sigma E_{0h}}{\gamma + \mu} + \left(I_{h0} - \frac{\sigma E_{h0}}{\gamma + \mu} \right) e^{-(\gamma + \mu)t} \tag{3.73}$$

$$R_h = \frac{\gamma I_{h0} + a S_{h0}}{\mu} + \left(R_{h0} - \frac{\gamma I_{h0} + a S_{h0}}{\mu} \right) e^{-\mu t} \tag{3.74}$$

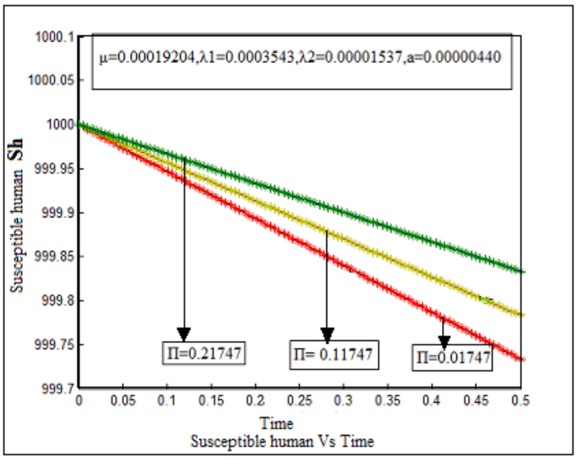
$$S_v = \frac{\pi_1}{\lambda_v + \mu_1 + b} + \left(S_{v0} - \frac{\pi_1}{\lambda_v + \mu_1 + b} \right) e^{-(\lambda_v + \mu_1 + b)t} \tag{3.75}$$

$$E_v = \frac{\lambda_v S_{v0}}{\sigma_1 + \mu_1 + b} + \left(E_{v0} - \frac{\lambda_v S_{v0}}{\sigma_1 + \mu_1 + b} \right) e^{-(\sigma_1 + \mu_1 + b)t} \tag{3.76}$$

$$I_v = \frac{\sigma_1 E_{v0}}{\mu_1 + b} + \left(I_{v0} - \frac{\sigma_1 E_{v0}}{\mu_1 + b} \right) e^{-(\mu_1 + b)t} \tag{3.77}$$

4. RESULTS AND DISCUSSIONS:

The analytical solution of the system of equations of the susceptible human population, the exposed human population, the infected human population, the recovered human population, the susceptible vector population, the exposed vector population, infected vector population (2.1) - (2.7) are given in equations (3.36) - (3.42) & (3.71) - (3.77) using homotopy perturbation & new homotopy perturbation respectively and is compared with its numerical simulation.



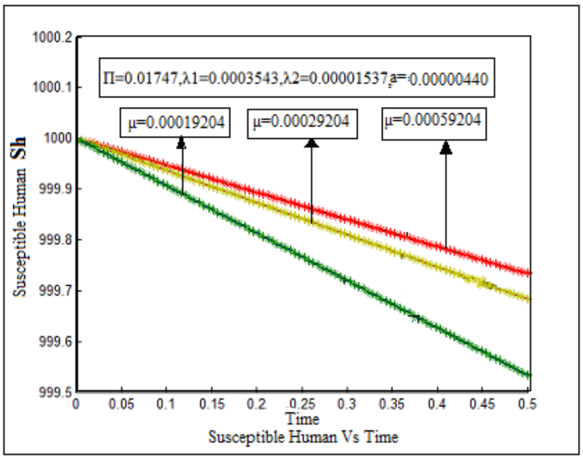
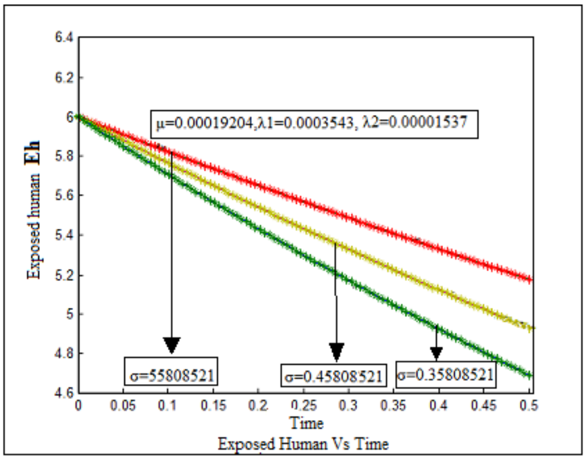


Figure 1(a) **Figure 2(b)**

Figure 1(a) and 1(b): *Figure 1(a) and 1(b) represent the susceptible human population* S_h *versus time t for the parameter, the recruitment rate of humans* **(π)** *and the natural death rate of humans* **(μ)**. *In the graph, the dashed and the star line represent the analytical solution* *equation of (HPM) & (NHPM) respectively and the dotted line represents the numerical solution.*

From Figures (1(a)) & (1(b)) see that the susceptible population rate versus time for the human population rate reduces for the lost values of the recruitment rate of humans (π) and natural death rate of human (μ) for the time and the other parameters are remain fixed.



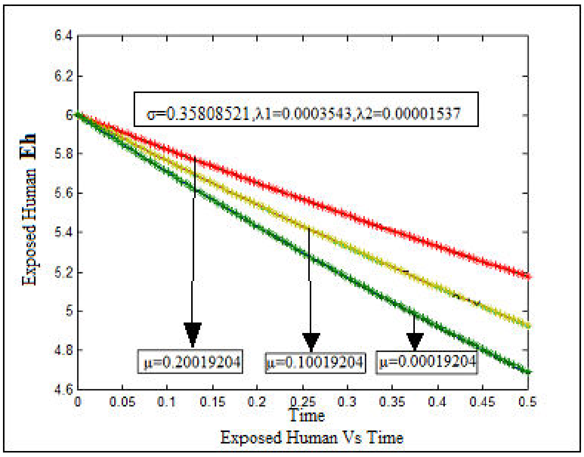
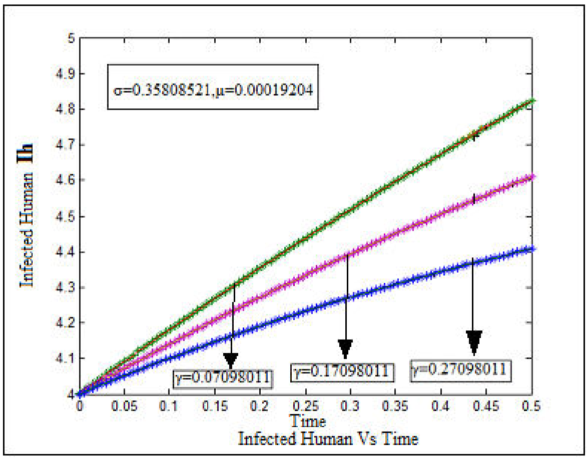


Figure 2(a) **Figure 2(b)**

Figure 2(a) and 2(b): *Figure 2(a) and 2(b) represent the exposed human population* $\boldsymbol{E}_h$ *versus time for the parameter, the progression rate from exposed to infected human (**σ**) and the natural death rate of humans (**μ**). In the graph, the dashed and the star line represent the analytical solution equation of (HPM) & (NHPM) respectively and the dotted line represents the numerical solution.*

From Figures (2(a)) & (2(b)) state that the exposed population rate versus time for the human population rate diminishes for the shrink values of progression rate from exposed to infected human (σ) and natural death rate of human (μ) for the time and the left out parameters are kept stable.



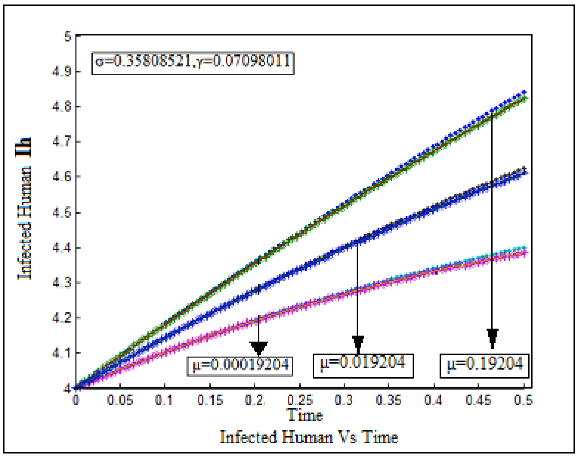
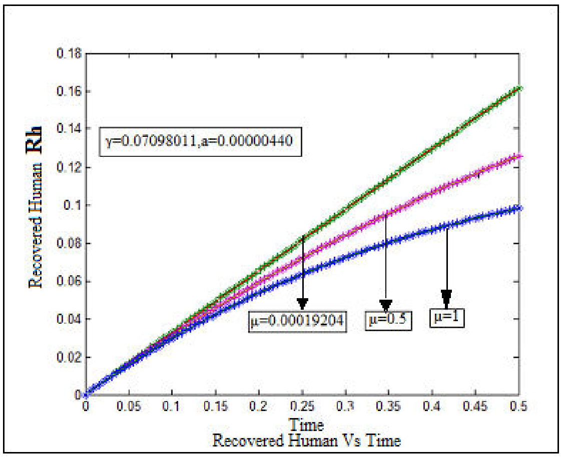


Figure 3(a) **Figure 3(b)**

Figure 3(a) and 3(b): *Figure 3(a) and 3(b) represent the infected human population* $\boldsymbol{I}_h$ *versus time for the parameter, the recovery rate of infected human (**γ**) and the natural death rate of* *human (**μ**). In the graph, the dashed and the star line represent the analytical solution equation of (HPM) & (NHPM) respectively and the dotted line represents the numerical solution.*

From Figures (3(a)) & (3(b)) observe that the infected population rate versus time for the human population rate increase for the decreased values of the recovery rate of infected humans (γ) and the increased values of natural death rate of human (μ) for the time and the leftover parameters are still static.



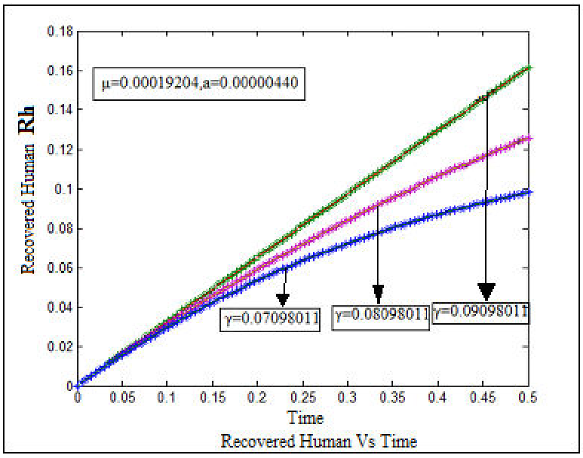
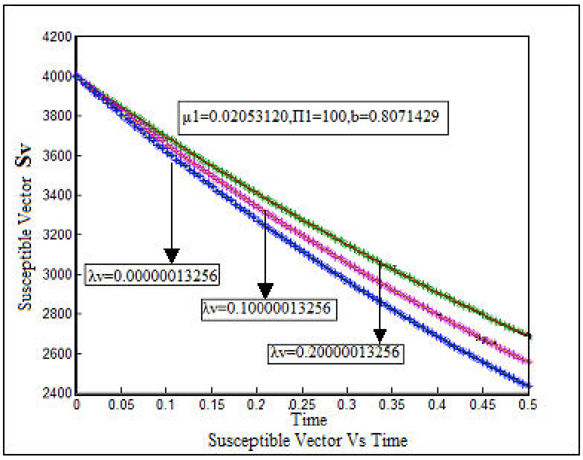


Figure 4(a) **Figure 4(b)**

Figure 4(a) and 4(b): *Figure 4(a) and 4(b) represents the recovered human population R_h versus time for the parameter, the natural death rate of humans (**μ) and** the recovery rate of infected humans (**γ**). In the graph, the dashed and the star line represent the analytical olution equation of (HPM) & (NHPM) respectively and the dotted line represents the numerical solution.*

From Figure (4(a)) notice that the recovered population rate versus time for the human population rate grows for the falling value natural death rate of humans (μ) for the time and γ & a stay constant. From Figure (4(b)) show that the recovered population rate versus time for the human population rate goes down for the dwindling value of recovery rate of infected human (γ) for the time and μ & a retain permanently.



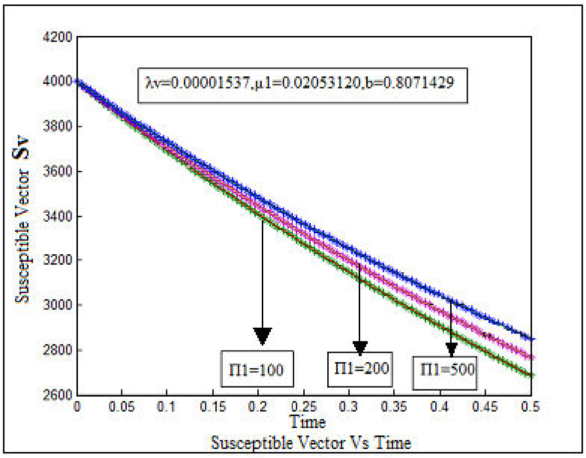


Figure 5(a) **Figure 5(b)**

Figure 5(a) and 5(b): *Figure 5(a) and 5(b) represents the susceptible vector population* $\boldsymbol{S}_v$ *versus time for the parameter, susceptible mosquitoes acquire infection from infected human* ($\boldsymbol{\lambda}_v$) *and the recruitment rate of mosquitoes* ($\boldsymbol{\pi}_1$)*. In the graph, the dashed and the star line represent the analytical solution equation of (HPM) & (NHPM) respectively and the dotted line represents the numerical solution.*

From Figure (5(a)) & (5(b)) describe that the susceptible vector population rate versus time for the mosquitoes population rate go narrow for the drop values of susceptible mosquitoes acquiring infection from infected human (λ_v) & recruitment rate of mosquitoes (π_1) for the time and residual parameters hold solid.

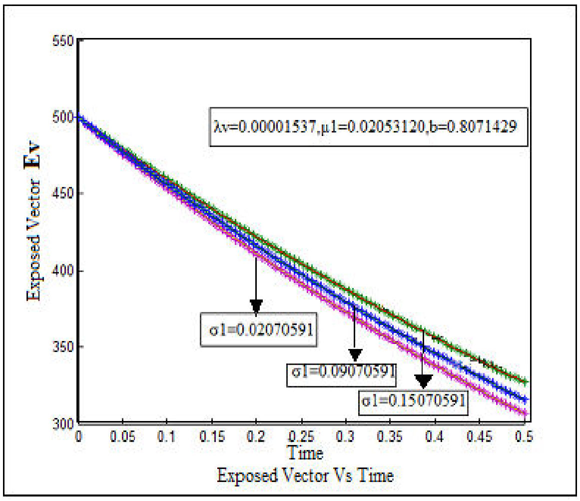


Figure 6(a)

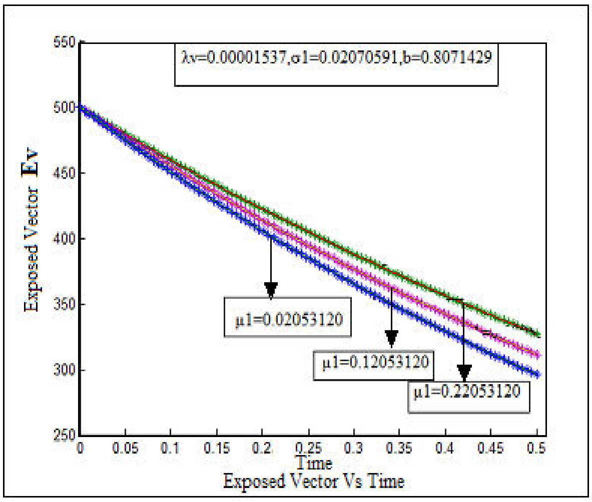
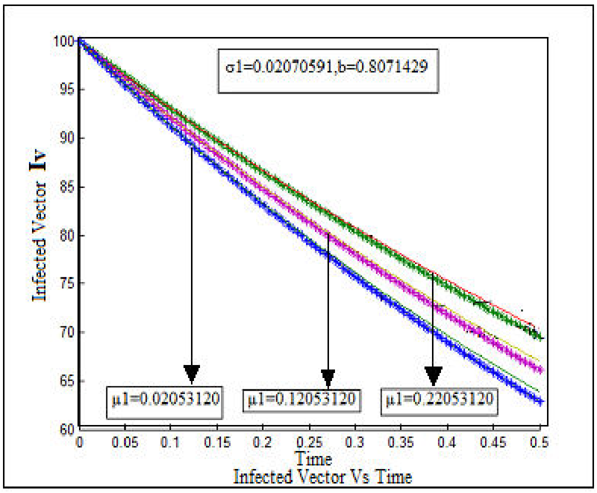


Figure 6(b)

Figure 6(a) and 6(b): *Figure 6(a) and 6(b) represents the exposed vector population* $\boldsymbol{E}_v$ *versus time for the parameter, the progression rate from exposed to infected mosquitoes* (***σ1***) *and the natural death rates of mosquitoes* (***μ1***)*. In the graph, the dashed and the star line represent the analytical solution equation of (HPM) & (NHPM) respectively and the dotted line represents the numerical solution.*

From Figures (6(a)) & (6(b)) sketch that the exposed vector population rate versus time for the mosquitoes population rate declined for the shortened values of progression rate from exposed to infected (σ_1) & natural death rate of mosquitoes (μ_1) for the time and rest parameters sustain rigidly.



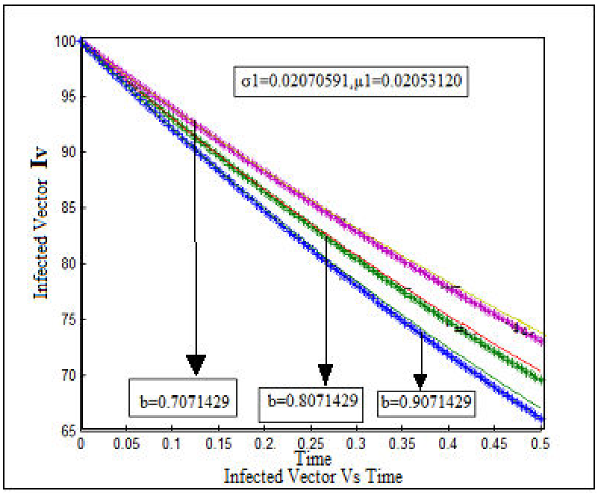


Figure 7(a) **Figure 7(b)**

Figure 7(a) and 7(b): *Figure 7(a) and 7(b) represents the infected vector population* I_v *versus time for the parameter, the natural death rate of mosquitoes* (***μ1)*** *and the constant rate of effective mosquito control* (***b).*** *In the graph, the dashed and the star line represent the analytical solution equation of (HPM) & (NHPM) respectively and the dotted line represents the numerical solution.*

From Figure (7(a)) tells that the infected vector population rate versus time for the mosquito population rate lessens for the lower value of natural death rate of mosquitoes (μ_1) for the time and b & σ1 reserve stationary. And Figure (7(b)) explains that the infected vector population rate versus time for the mosquito population rate plunge for the rising value of the constant rate of effective mosquito control (b) for the time and μ1 &σ1 parameters perpetuate steady.

5. CONCLUSION:

Thus the system of non-linear differential equations on the suspectable human population, exposed human population, infected human population, recovered human population, suspectable vector population, exposed vector population and infected vector population have been solved using the Homotopy Perturbation method (HPM) & New Homotopy Perturbation method (NHPM) and the meticulous of the approximate analytical solution has been verified by comparison with its numerical simulation. Thus the analytic result helps us to understand the effect of various parameters on the Zika virus model.

6. ACKNOWLEDGEMENT

The author(s) declare that they have not received any fund or grant from any institution for their research work.

7. CONFLICTS OF INTEREST

Both the author(s) confirm that they have no conflicts of interest.

REFERENCES :

[1] Agusto, F. B., Bewick, S., & Fagan, W. F. “Mathematical model for Zika virus dynamics with Sexual transmission route Ecological Complexity”, (2017a), Vol. 1, pg. 61-81.

[2] AgustoF.B., S. Bewick, and W.F. Fagan.**“**Mathematical model of Zika virus with vertical transmission. Infectious Disease Modelling’ 2017, Vol. 1, Pg. 1-24.

[3] Aranda, L. D. F., Gonzalez-Para, G., &Benincasa, T. “Mathematical modelling and numerical simulations of Zika in Colombia considering mutation”(2019), Vol. 1, pg. 1-18.

[4] BearcroftW.G.C, Zika virus infection experimentally induced in human volunteer Transactions of the Royal Society of Tropical Medicine & Hygiene”, 50 (5) (1956),Vol 1, pg. 442-448.

[5] Bonyah, E., Khan, M. A., Okosun, K. O., & Islam, S. A. “Theoretical Model for Zika virus transmission”PLoS One, 12 (10) (2017),Vol. 1, Article e0185540.

[6] Dallas County Health Human Services (DCHHS) (2 February 2016). DCHHS reports “first zika virus case in Dallas country acquired through sexual transmission”.

[7] Dick, G.W., Kitchen, S. F., &Haddow, A. J. Zika virus (I)“Isolations and serological specificity. Transmission of the Royal Society of Tropical Medicine and Hygiene”46(1952), Vol.1, pg. 509-520.

[8] Duffy, M. R. “Zika virus outbreak on Yap Island, Federated States OF Micronesia”,360(2009), Vol. 1,pg. 2536-2543.New England Journal of Medicine.

[9] Foy, B. D. “Probable non-vector-borne transmission of zika virus, Colorado, USA Emerging Infectious Diseases”17(2011), Vol. 1,pp. 880-882.

[10] Gao D, Y. Lou, D. He, T.C Porco, Y. Kuang, G Chowell The Toronto Star, (2016) Canada’s first case of sexually-transmitted Zika virus confirmed in Ontario, April 25,2016.

[11] Grajales-MuñizC, VH Borja-Aburto Zika-epidemiological report. (25 September 2017). Colombia: PAHO. WHO

[12] HancockW.T. , M. Marfel, M. Bel, Zika virus ,French Polynesia, south pacific, 2013 Emerging Infectious Diseases, 20 (2014), Vol. 1, pg. 1085-1086.

[13] HeangV, CY Yasuda, L Sovann, European Centre for Disease Control and Prevention (CDC).Zika virus infection. Factsheet for Health Professionals (2015) , Vol.1, 18(2), pp. 349–351.

[14] Imran, M., Usman, M., Dur-e-Ahmad, M., & Khan, A. Transmission dynamics of zika fever: A SEIR based model & Differential Equations and Dynamical (2017), Vol.1, Ecol. Complex. **29**, 1–92.

[15] Kucharski, A. J., Funk, S., Eggo, R. M., Mallet, H., Edmunds, W. J., &Nilles, E. J. (2016).Transmission dynamics of zika virus in Island population, A modelling analysis of the 2013-2014 French Polynesia outbreak”, Neglected Tropical Diseases, 10(5) 2014.

[16] MacnamaraF.N. ,Zika virus: A report on three cases of human infection during an epidemic of jaundice in Nigeria**,** Transactions of the Royal Society of Tropical Medicine & Hygiene”, 48 (2) (1954), Vol. 1, pg. 139-145.

[17] Manore C, and Hyman M, “Mathematical models for fighting Zika virus”, SIAM news, Retrieved May 2, 2016.

[18] MlakarJ. ,Zika virus associated with microcephaly, New England Journal of Medicine, 374 (10) (2016), Vol. 1, pg. 951-958.

[19] Musso, D. "Potential sexual transmission of Zika virus &Emerging Infectious Diseases", Vol.1, 21 (2) (2015), pg. 359-361.
[20] Nicole J. OlynkWidmar, S.R. Dominick, Audrey Ruple, and Wallace E. Tyner. "The inuence of health concern on travel plans with focus on the Zika virus in 2016". Preventive Medicine Reports **6**. 2017. Vol. 1, Pg. 162–170.
[21] Oster. AM – MMWR, "Centres for Disease Control and Prevention (CDC)" (23 February 2016). Update: "Interim guidelines for prevention of sexual transmission of zikavirus",United States, 2016.
[22] SikkaV. , V.K. Chattu, R.K. Popli, The emergence of zika virus as global health security threat: A new and a consensus statement of the INDUSEM joint working group (JWG) Journal of Global Diseases", 8 (1) (2016), Vol. 1, pg. 3-15.
[23] YitadesGebre, Nikkiah Forbes, and TeshomeGebre. "Zika virus infection, transmission, associated neurological disorders and birth abnormalities: A review of progress in research, priorities and knowledge gaps". Asian Pac J Trop Biomed. 2016. Vol. 1, **6**(10):
Pg. 815–824
[24] Shah, N. H., Patel, Z. A., &Yeolekar, B. M. "Prevention and controls on congenital transmissions of zika: Mathematical analysis &Applied Mathematics", Vol. 1,8 (2017), pg. 500-519.
[25] Sudhanshu Kumar Biswas, Uttam Ghosh, Susmita Sarkar, " Mathematical model of zika virus dynamics with vector control and sensitivity analysis", Infectious Disease Modelling, 5(2020) pg23-41.

CONNECTEDNESS AND COMPACTNESS ON TSBF-ALGEBRAS

M. Jansi

Guest Lecturer, Department of Mathematics,
Government Arts and Science College,
Nagalapuram, Tamil Nadu, India
E-Mail: mjansi92@gmail.com

ABSTRACT: *Connectedness and compactness are useful and fundamental notions. Connectedness is one of the principal topological property that are used to distinguish topological spaces. Compactness is a way to generalize the properties of finite sets to more general sets. In this paper we define these two fundamental concepts on TSBF-algebras and prove some of properties on it.*

1. INTRODUCTION:

In this paper, we define two fundamental concepts compactness and connectedness on TSBF−algebras in more easier way with the help of identity element of a BF−algebra. We give the simpler form of the definitions of compactness and connectedness. Also we discuss the necessary condition for separation axioms T_0, T_1 *and* T_2 to hold on TSBF−algebras.

2. PRELIMINARIES :

Definition 2.1. [2] A BF-algebra is an algebra (X,∗,0) of type (2,0) satisfying the following conditions:

1. x∗x = 0.
2. x∗0 = x.
3. 0∗(x∗y) = y∗x,∀ x,y ∈ X.

Definition 2.2.[9] Let (F,A) be a soft BF-algebra over X and τ be a soft BF− topology on X. Let x ∈ A. Then (F,A,τ) is said to be a topological soft BF-algebra over X with respect to F(x), if for every a,b ∈ F(x) and any open set W of a∗b, there exist open sets U and V of a and b respectively such that U ∗V ⊆ W. That is, a function f from F(a)×F(a) into F(a) is continuous with respect to a topology τ on X, since F(a) is a subalgebra of X.

Definition 2.2.1 [9] Let (F,A,τ) be a topological soft BF -algebra (TSBF -algebra) over a BF – algebra X with respect to F(a),a ∈ A Then (F,A,τ) is said to be a topological soft BF1 algebra (TSBF1 -algebra) over X with respect to F(a), if x = (x∗y)∗(0∗y), for all x,y ∈ X.

Theorem 2.3.[8] Let (F,A,τ) be a TSBF−algebra with respect to F(a) over X, and $\{G_\alpha\}$α∈I be the collection of open sets contained in F(a), I ia an index set. Then, arbitrary intersection of open sets Gα 's is open.

Theorem 2.4.[10] Let (F,A,τ) be a TSBF1 -algebra with respect to F(a) over X. Then the smallest open set for 0, is a subalgebra of X.

3. CONNECTEDNESS :

Definition 3.1

Let (F,A,τ) be a $TSBF_1-$ algebra with respect to F(a) over X, and S ⊆ X. Then S is said to be separated if S = A∪B, where A and B are the non-empty disjoint open sets in X. If S is not separated then S is connected.

Remark 3.2

1. If U&V are connected subsets of X then U ∗V is also connected.
2. The left map l_a, a ∈ F(a) maps connected sets into connected sets.

Theorem 3.3. Let (F,A,τ) be a $TSBF_1-$ algebra with respect to F(a) over X. Then,

1. If 0 is an interior point of F(a), then F(a) is separated.
2. If 0 is not an interior point of F(a), then F(a) is connected and every subset S of F(a) is also connected.
3. If 0 is not an interior point of F(a), then (the least open set of 0) L_0 is connected.

Proof: (1). Assume 0 is an interior point of F(a). So, every x ∈ F(a) is an interior point of F(a). Then F(a) can be written as finite union of open sets.

=⇒ F(a) = $\cup_{\{x \in F(a)\}} L_x = L_0 \ \cup_{\{x \in F(a)-L_0\}} L_x$. Therefore, F(a) is separated.

(2) Assume 0 is not an interior point of F(a). Int F(a) = φ. Implies, there is no open set contained in F(a)...(1). Therefore, F(a) cannot be separated into union of two disjoint open sets....(2). Hence, F(a) is connected.

(3). Now, (1) & (2) is true for every S ⊆ F(a) and L_0. So, S and L_0 is connected.

Theorem 3.4. Let (F,A,τ) be a $TSBF_1-$ algebra with respect to F(a) over finite X. Then, F(a) is connected if 0 ∈ $\cap_{\{U_\alpha \in \tau-\{\varphi\}\}}$Uα.

Proof: Since 0 belongs to all open sets U_α of X except φ and from lemma 3.1.16, we have F(a) ⊆ U_α, for all U_α ∈ τ −{φ}. So, F(a) cannot be written as disjoint union of two non-empty open sets contained in F(a). Hence, F(a) is connected.

Theorem 3.5. Let (F,A,τ) be a $TSBF_1-$ algebra with respect to F(a) over finite X with the condition 0 ∗ x = x, for all x ∈ X and let 0 be an interior point of F(a). If (m,n) = 1 where o(F(a)) = n and o(S) = m, then S is connected where S ⊆ F(a).

Proof: Let S ⊆ F(a). Then, o(F(a)) and o(L_0) is even. o(L_x) is even.

Since (m,n) = 1, then o(S) is odd. Suppose S is not connected. Then, S = A ∪ B, where A and B are disjoint non-empty open sets. Since X is finite, the smallest open set for every x ∈ X is open.

Therefore, S = $(\cup_{\{x \in A\}} L_x) \cup (\cup_{\{x \in B\}} L_x) = \cup_{\{x \in A \cup B\}} L_x = \cup_{\{x \in S\}} L_x$. Then, o(S) =$\cup_{\{x \in S\}} o(L_x$)is even, which is a contradiction to the fact that o(S) is odd. Hence, S is connected.

Theorem 3.6. Let (F,A,τ) be a $TSBF_1-$ algebra with respect to F(a) over X, with the condition that 0∗x = x, ∀x ∈ X. If there exist an open ideal contained in F(a), then F(a) is not connected.

Proof: Let I be an open ideal contained in F(a) and x ∈ (F(a)−I) Since 0 ∈ I & x∗x = 0, there exist open sets U and V of x such that, U ∗V ⊆ I...(1) If V ⊆ (F(a)−I) then F(a) is separated. If not, there exist an element say y, y ∈ V and y / ∈ (F(a)−I).

Case(i): Let y ∈ I^c and y ∉ F(a). (1) implies, x∗y ∈ F(a). From lemma 2.2.12, y ∈ F(a). Therefore, y ∈ F(a) for all y ∈ V.

Case(ii): Let y ∉ I^c and y ∈ F(a). So, y ∈ I. (1) implies, x∗y ∈ I. Since I is ideal, x ∈ F(a). This is a contradiction. Therefore, in both cases we can conclude that, y ∈ F(a)−I for all y ∈ V.

Theorem 3.7. Let (F,A,τ) be a TSBF− algebra with respect to F(a) and S ⊆ F(a). S is disconnected if and only if S = $\cup_{\{x\in S\}} L_x$.

Proof: Let S be any subset of F(a). Assume, S = $\cup_{\{x\in S\}} L_x$. Clearly, $L_x \cap L_y$ = φ,∀ x,y ∈ F(a). S = $L_x \cup \left(U_{\{y\in S-L_x\}} L_x \right)$. Therefore, S is disconnected.

Conversely, assume S is disconnected.

⇒ S = A∪B, where A and B are open and A∩B = φ, B∩A = φ. Clearly, A & B ⊆ F(a). Implies, the smallest open set for every x in A, is contained in A. So, A = $\cup_{\{x\in A\}} L_x$. Similarly, we can write, B = $\cup_{\{x\in B\}} L_x$. Therefore, S = $(\cup_{\{x\in A\}} L_x) \cup (\cup_{\{x\in B\}} L_x) = \cup_{\{x\in A\cup B\}} L_x = \cup_{\{x\in S\}} L_x$.

Theorem 3.8. Let (F,A,τ) be a $TSBF_1$− algebra with respect to F(a) over X, with the condition that 0∗x = x, ∀ x ∈ X. If B ⊆ F(a) is connected then the image of B under the left map restricted to F(a) l_p, p ∈ F(a) is connected.

Proof: The proof of this theorem follows from continuous image of connected set is connected.

Theorem 3.9. Let (F,A,τ) be a $TSBF_1$− algebra with respect to F(a) over X, with the condition that 0∗x = x, ∀ x ∈ X. If S ⊆ F(a) is separated then the image of S under the left map l_p, $p \in F(a)$ is separated.

Proof: Let S be a separated set in X. Then S = A∪B, where A amd B are disjoint open subsets of X. Now, l_p (S) = p∗S. Now, p∗S = p∗(A∪B) = (p∗A)∪(p∗B). Since A and B are open, p∗A and p∗B is open. Since A∩B is empty, we have (p∗A)∩(p∗B) = φ. Therefore, the image of S under the left map l_p, p ∈ F(a) is separated.

4. COMPACTNESS :

Definition 4.1.

Let (F,A,τ) be a TSBF−algebra with respect to F(a) over X and C ⊆ X. A collection A of subsets of a space X is said to cover C, or to be a covering of C, if the union of the elements of A is equal to C. It is called open covering of C if its elements are open subsets in X.

Definition 4.2. Let (F,A,τ) be a TSBF−algebra with respect to F(a) over X. A space X is said to be compact if every open covering A of X contains finite subcollection that also covers X.

Theorem 4.3. Let (F,A,τ) be a TSBF−algebra with respect to F(a) over X and C ⊆ F(a) is compact. Then image of C under a left map is compact.

Proof : Since for every left map l_a, a ∈ F(a) restricted to F(a) is continuous, la(C) is compact.

Theorem 4.4. Let (F,A,τ) be a TSBF−algebra with respect to F(a) over X and C ⊆ F(a) is compact if and only if $C = \cup_{i=1}^{n} L_{x_i}$.

Proof: Assume, C is compact. Since for every x ∈ C, the smallest open set L_x is open, the collection L = {L_x / x ∈ C} of smallest open sets covers C. Therefore, the collection L has a finite sub cover say $L_{x_1}, L_{x_2}, \ldots, L_{x_n}$ that also covers C. Hence, $C \subseteq \cup_{i=1}^{n} L_{x_i}$.

Conversely, assume, $C \subseteq \cup_{i=1}^{n} L_{x_i}$. Let 𝔸 be an open covering of C. Let x ∈ C, there is at least one element say $A_1 \in$ 𝔸 Clearly, the smallest open set $L_x \subseteq A_1$. For every x ∈ C, there is an element $A_x \in$ 𝔸 and $L_x \subseteq A_{x_i}$. Since $C \subseteq \cup_{i=1}^{n} L_{x_i} \subseteq \cup_{i=1}^{n} A_{x_i}$. Implies, $C \subseteq \cup_{i=1}^{n} A_{x_i}$. Therfore, the open cover A has finite subcover that covers C. Hence, C is compact.

Theorem 4.5 Let (F,A,τ) be a $TSBF_1$−algebra with respect to F(a) over X with the property that, 0 ∗ x = x,∀ x ∈ X and $C_1, C_2, \ldots, C_n \subseteq F(a)$ are compact. Then $C_1 * C_2 * \ldots * C_n$ is compact.

Proof: We want to prove this theorem by induction on n.

Let n = 2. Let $C_1, C_2 \subseteq$ F(a) are compact. From theorem 2.4, $C_1 \subseteq \bigcup_{i=1}^{n} L_{x_i}$and $C_2 \subseteq \bigcup_{i=1}^{m} L_{y_i}$, where $x_i \in C_1$ and $y_i \in C_2$. Let $z \in C_1 * C_2$. So, z = x∗y, where x ∈ C_1 and y ∈ C_2. $L_z = L_{\{x*y\}} = L_x * L_y$. Implies, $L_{x_i} * L_{y_i} = L_{z_i}$. Let k = max{n,m}. Therefore, $C_1 * C_2 \subseteq \bigcup_{i=1}^{k} L_{z_i}$, where L_{z_i} is the smallest open set for $z_i \in C_1 * C_2$. From theorem, 2.4, $C_1 * C_2$ is compact.....(1). Assume, $C_1 * C_2 * \ldots * C_{n-1}$ is compact........(2). For n, $C_1 * C_2 * \ldots * C_n = \left(C_1 * C_2 * \ldots C_{\{n-1\}}\right) * C_n$ is compact.

Remark 4.6.

1. Finite union of compact set is compact.
2. Arbitrary intersection of compact set is compact.

Theorem 4.7. Let (F,A,τ) be a $TSBF_1$−algebra with respect to F(a) over X with the property that, 0 ∗ x = x,∀ x ∈ X. Arbitrary union of compact sets contained in F(a) is compact.

Proof: From theorem 3.17 and theorem 2.3 we can prove this theorem.

Remark 4.8.

In above theorem 4.7, put F(a) = X, then arbitrary union of compact subsets of X is compact.

Theorem 4.9. Let (F,A,τ) be a $TSBF_1$−algebra with respect to F(a) over X. If 0 is not an interior point of F(a), then every subset of F(a) is compact.

Proof: Assume, 0 is not an interior point of F(a). From theorem 4.7, there is no interior points of F(a). Let A ⊆ F(a). Therefore, every open cover of A has a finite sub cover that also covers A. A is compact.

Theorem 4.10. Let (F,A,τ) be a $TSBF_1$−algebra with respect to F(a) over X and B ⊆ F(a). If IntB = φ, then B is compact.

Proof: Let B ⊆ F(a). Since IntB = φ, 0 is not an interior point of B and F(a).

From theorm 4.9, B is compact.

Theorem 4.11. Let (F,A,τ) be a $TSBF_1$−algebra with respect to F(a) over X and 0 belongs to every open set of $x \in A$. Then every subset of F(a) is compact.

Proof: Let B ⊆ F(a). , IntB = φ.

From theorem 4.9, B is compact.

Remark 4.12.

1. The smallest open set for x ∈ F(a) is compact.
2. Every open subspace of compact space is compact.
3. Compact sets are separated.
4. If 0 is not a limit point of a set S ⊆ F(a), then S is compact.
5. Finite cross product of compact sets is compact.

REFERENCES :

[1] Arhangelski, M. Tkachenko, Topological groups and related structures, Atlantis press, 2008.

[2] Andrzej Walendziak: On BFalgebras, Mathematical Solvaca, math. Solvaca 57 (2007), No.2, 119-128.

[3] Borzooei R. A, Rezaei G.R and kouhietani k :On(semi)topological BLalgebras, Inanian journal of math,sci and vol 6(2011).
[4] Baumslag. G, Myasnikov. A, Remeslennikov. V, Algebraic geometry over groups. I. Algebrais sets and ideal theory, J.Algebra 219, (1999) 16-79.
[5] Bryant. R, THe Verbal Topology of a group, Journal of Algebra, 48 (1977) 340-346.
[6] CHANG C. L, Fuzzy topological spaces, Journal of Mathematics Analysis Application 24(1968) 182-190.
[7] James R. Munkers: Toplogy, second edition, Pearson Education, Inc.,
[8] Jansi M and Thiruveni V: Complementry Role odf Ideals in TSBF−algebras, Malaya Journel of Mathematik, Vol. S, No.1, 2019, 8 3-85.
[9] Jansi . M and Thiruveni. V: Topological Soft BF-Algebras, Compliance Engneering Journal, Volume 11, Issue1, 2020.
[10] Jansi . M and Thiruveni. V: A Note on TSBF1−algebras, Indian Journal of Natural Sciences, Vol.12, Issue 69, December 2021.
[11] Jansi . M and Thiruveni. V: Topological BCH−groups, Malaya Journal of Mathematik, Vol.S, No.1, 83-85, 2019**.**
[12] Jansi M and Thiruveni V: Topological Structures on BCH-algebras, IJIRSET, Vol.6, 2017, 22594 - 22600.
[13] Jansi . M and Thiruveni. V: Ideals and Separation Axioms in LSTBCH−Algebra, International Journal of Mathematics Trends and Technology, Vol.60, No.2, Augest, 2017.
[14] Jun Y. B and Park C. H : Applicatons of Soft Sets in Ideal Theory of BCK/BCI Algebras, Inform, Sci. 178(2008)2466-2475.

Anti multi fuzzy BH-ideals in BH-algebras

K. Anitha

Assistant Professor, Department of Mathematics,
Sri Krishnasamy Arts and Science College, Sattur - 626 203, Tamil Nadu,
India. Mail id:anithamaths19@gmail.com

ABSTRACT: *In this paper we speak approximately the Anti multi fuzzy BH-Ideals and associated homes in BH-Algebras. Multi Fuzzy set concept is a extension of fuzzy set concept. These offers with the multi-dimensional fuzziness. we introduce the perception of Anti multi-fuzzy BH-ideals, the Anti multi-level subset of BH-ideals. And additionally we outline a few associated Anti multi-fuzzy BH-ideals based on level subset of it.*

KEYWORDS: *BH-algebra, Anti Fuzzy BH-ideal, Anti Multi-fuzzy BH-ideal. Anti Multi fuzzy closed ideal.*

Subject Classification*: AMS (2000), 06F35, 03G25, 06D99, 03B47*

1. INTRODUCTION:

Y. Imai and K. Iseki [1,2&3] are brought lessons of summary algebras. BCK- algebras and BCI-algebras. It is understood that the elegance of BCK-algebras is a right subclass of th elegance of BCI-algebras. K. Iseki and S. Tanaka [4] are brought creation to concept of BCK-algebras. L.A. Zadeh [5] are brought fuzzy units. S. Sabu and T.V. Ramakrishnan[6] are brought Multi-Fuzzy units, The preception of BH-algebras is brought with the aid of using J.B. Jun, E.H. Roh and H.S. Kim[7] .Since then, numerous authors have studied BH-algebras. In particular, Q. Zhang, E.H. Roh and Y.B. Jun [8] studied the fuzzy concept in BH-algebras. K. Anitha and N. Kandaraj [9] are brought Fuzzy subalgebras on BH-algebras. K. Anitha and N. Kandaraj are brought Fuzzy ideals and Fuzzy dot ideals on BH-algebras. In this paper, we outline Anti multi-fuzzy ideals in BH-algebra and talk a number of their associated primarily based on level subsets and homomorphism.

2. PRELIMINARIES:

In this phase we talk the fundamental definitions of a BH-algebras.

Definition 2.1:[1,2,3] Let X be a nonempty set with a binary operation $*$ and a constant 0. Then (X, *, 0) is referred to as a BCI-algebras if it satisfies the subsequent conditions.

1. $((x*y)*(x*z))*(z*y)=0$
2. $(x*(x*y))*y=0$
3. $x*x=0$
4. $x*y=0$ and $y*x=0 \Longrightarrow x=y \;\; \forall x,y \in X.$

Example 2.2: Let $X=\{0,a,b,c\}$ be a set with the subsequent cayley table.

*	0	a	b	C
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
c	c	c	c	0

Then $(X,*,0)$ is known as a BCI-algebras.

Definition 2.3:[1,2,3] Let X be a nonempty set with a binary operation $*$ and a constant 0. Then (X, *, 0) is known as a BCK-algebras if it satisfies the subsequent conditions.

1. $\big((x*y)*(x*z)\big)*(z*y)=0$
2. $\big(x*(x*y)\big)*y=0$
3. $x*x=0$
4. If $x*y=0$ and $y*x=0 \Longrightarrow x=y$
5. $0*x=0$ for all $x,y,z\in X$.

Example 2.4: Let$X=\{0,1,2,3\}$ be a set with the subsequent cayley table.

*	0	1	2	3
0	0	0	0	0
1	1	0	1	2
2	2	3	0	0
3	3	1	2	0

Then $(X,*,0)$ is known as a BCK-algebras.

Definition 2.5:[7,8] Let X be a nonempty set with a binary operation $*$ and a constant 0. Then $(X,*,0)$ is referred to as a BH-algebras if it satisfies the following conditions.

1. $x*x=0$
2. $x*0=x$
3. If $x*y=0$ and $y*x=0 \Longrightarrow x=y$ $x,y\in X$.

Example 2.6: Let $X=\{0,1,2,3\}$ be a set with the subsequent cayley table.

*	0	1	2	3
0	0	1	2	3
1	1	0	2	1
2	2	3	0	0
3	3	2	3	0

Then $(X,*,0)$ is known as a BH-algebras.

Definition 2.7:[8]

Let S be a nonempty subset of a BH-algebra X, then S is referred to as subalgebra of BH-algebra if $x*y\in S$ for all $x,y\in S$.

Definition 2.8:[8]

Let X be a BH-algebra and I be a subset of X, then I is known as a ideal of X if Satisfies the following conditions.

1. $0\in I$
2. $x*y\in I$ and $y\in I \Longrightarrow x\in I$ for all $x,y\in I$
3. $x\in I$ and $y\in X \Longrightarrow x*y\in I$

Definition 2.9:[9]
Let σ be a fuzzy set in a BH-algebra X. Then σ is referred to as a fuzzy BH-subalgebra if $\sigma(x * y) \geq \min\{\sigma(x), \sigma(y)\}\ \forall x, y \in X$

Definition 2.10:[7,8,10]
Let σ be a fuzzy set in a BH-algebra X. Then σ is referred to as a fuzzy BH-ideal if it satisfies the subsequent conditions.

1. $\sigma(0) \geq \sigma(x)$
2. $\sigma(x) \geq \min\{\sigma(x * y), \sigma(y)\}$
3. $\sigma(x * y) \geq \min\{\sigma(x), \sigma(y)\}\ \forall x, y \in X.$

Definition 2.11[7,8] A mapping $g: X \to Y$ of a BH-algebra is referred to as a homomorphism if $g(x * y) = g(x) * g(y)\ \forall x, y \in X.$

Definition 2.12[6]
Let X be a nonempty set. Define a multi-fuzzy set B in X is a set of ordered sequences: $B = \{(x, \sigma_1, \sigma_2, \dots \dots \sigma_i \dots \dots): x \in X\}$, where $\sigma_i: X \to [0,1]$ for all i

Remark 2.13[6]

1. If the sequences of the membership functions have only k-terms(finite wide of terms) k is called the dimension of B.
2. The set of all multi-fuzzy sets in X of dimension k is denoted through $M^kFS(X)$.
3. The multi-fuzzy membership function $\sigma_B(x)$ is a function from X to $[0,1]^k$such that for all $x \in X$ $\sigma_B(x)$ =$(\sigma_1(x), \sigma_2(x), \dots, \sigma_k(x))$
4. For the sake of simplicity, we denote the multi-fuzzy set as $B = \{(x, \sigma_1(x), \sigma_2(x), \dots \dots \sigma_k(x) \dots \dots): x \in X\}$ as $B = (\sigma_1, \sigma_2, \dots \dots \sigma_k)$.

Definition 2.14[6]
Let k be a positive integer and allow B and C in $M^kFS(X)$, where $B = (\sigma_1, \sigma_2, \dots \dots \sigma_k)$ and $C = (\rho_1, \rho_2, \dots \dots \rho_k)$ then we have got the subsequent members of the relations and operations:

1. $B \subseteq C$ if and only if $\sigma_i \leq \rho_k$, for all $i = 1,2, \dots \dots, k$
2. $B = C$ if and only if $\sigma_i = \rho_k$, for all $i = 1,2, \dots \dots, k$
3. $B \cup C = (\sigma_1 \cup \rho_1, \dots \dots \sigma_k \cup \rho_k) = \{(x, \max(\sigma_1(x), \rho_1(x)), \dots \dots \max(\sigma_k(x), \rho_k(x))): x \in X\}$
4. $B \cap C = (\sigma_1 \cap \rho_1, \dots \dots \sigma_k \cap \rho_k) = \{(x, \min(\sigma_1(x), \rho_1(x)), \dots \dots \min(\sigma_k(x), \rho_k(x))): x \in X\}$.

Definition 2.15[6]
Let B be a multi-fuzzy set in BH-algebra X. For any $s = (s_1, s_2, \dots \dots, s_k)$ where $s_i \in [0,1]$ for all i, the set $\cup\ (B; s) = \{x \in X / B(x) \geq s\}$ is referred to as the multi-level subset of B.

Definition 2.16[6]
Let B be a multi-fuzzy set in BH-algebra X. Then B is referred to as Anti multi-fuzzy closed ideal in X if it satisfies the subsequent conditions

1. $B(x) \leq \max\{B(x * y), B(y)\}$
2. $B(0 * x) \leq B(x)$

Example 2.17: Let $X = \{0,1,2,3\}$ be a set with the subsequent cayley table.

*	0	1	2	3
0	0	1	2	3
1	1	0	1	1
2	2	2	0	2
3	3	2	2	0

Then B is known as Anti multi-fuzzy closed ideal in X.

Definition 2.18[6]

Let σ be a fuzzy set in a BH-algebra X. Then σ is referred to as Anti fuzzy BH-ideal if it satisfies the subsequent conditions.

1. $\sigma(0) \leq \sigma(x)$
2. $\sigma(x) \leq \max\{\sigma(x * y), \sigma(y)\}$
3. $\sigma(x * y) \leq \max\{\sigma(x), \sigma(y)\} \, \forall x, y \in X$

3.ANTI MULTI-FUZZY BH-IDEAL IN BH-ALGEBRAS :

In this segment we mentioned the Anti multi-fuzzy BH-ideal and its properties.

Definition 3.1[6]

Let B be a multi-fuzzy set in BH-algebra X. Then B is known as a multi-fuzzy BH-ideal in X if it satisfies the subsequent conditions.

1. $B(0) \leq B(x)$
2. $B(x) \leq \max\{B(x * y), B(y)\}$
3. $B(x * y) \leq \max\{B(x), B(y)\} \, \forall x, y \in X$

Example 3.2: Let $X = \{0,1,2\}$ be a set with the subsequent cayley table.

*	0	1	2
0	0	1	2
1	1	0	1
2	2	2	0

Define a multi-fuzzy set $B: X \to [0,1]$ with the aid of using B(0)=B(1)=(p_1, p_2) and B(2)=(q_1, q_2) where p_1, p_2, q_1, $q_2 \in [0,1]$ with $p_1 > q_1$ and $p_2 > q_2$. Then B is Anti multi-fuzzy BH-ideal in BH-Algebras.

Theorem 3.3

Let X be a BH-algebra. Then B is Anti muti-fuzzy BH-ideal in X if and only if B is a Anti multi-fuzzy subalgebra of X.

Proof:

Let X be a BH-algebra.

Let B be Anti multi-fuzzy BH-ideal in BH-algebra X.

To show that B is Anti multi-fuzzy subalgebra in BH-algebra X

We recognize that Every Anti multi fuzzy BH-ideal of a BH-algebra X is a Anti multi- fuzzy subalgebra of X.

Let B be Anti multi fuzzy subalgebra in X.

To show that B is Anti muti fuzzy BH-ideal in X.

Let $x, y \in X$

By the use definition of BH-algebras conditions.

1) $B(0) = B(x * x)$
   $\leq \max\{B(x), B(x)\}$
   $B(x)\ \forall x \epsilon X$
2) $B(x) = B\big((x * y) * (0 * y)\big)$
   $\leq \max\{B(x * y), B(0 * y)\}$
   $\leq \max\{B(x * y), \max\{B(0), B(y)\}\}$
   $\leq \max\{B(x * y), B(y)\}$
3) It is in reality true.

Hence B is a Anti multi fuzzy BH- ideal in X.

Theorem 3.4:

Let B_1 and B_2 be two Anti multi fuzzy BH-ideals of a BH-algebra X. Then $B_1 \cup B_2$ is a Anti multi-fuzzy BH-ideal of X.

Proof:

Let B_1 and B_2 be two Anti multi fuzzy BH-ideals of a BH-algebra X.

To show that $B_1 \cup B_2$ is a Anti multi-fuzzy BH-ideal of X.

Let $x, y \in B_1 \cup B_2$.

Then $x, y \in B\ _1$ and $x, y \in B_2$

By the usage of multi fuzzy set union definition conditions

1. $B_1 \cup B_2(0) = (x * x)$
   $B_1 \cup B_2 = \max\{\, B_1(x * x), B_2(x * x)\}$
   $\leq \max\{\max\{B_1(x), B_1(x)\}\}.\ \max\{B_1(x), B_2(x)\}\}$
   $= \max\{B_1(x), B_2(x)\}$
   $= B_1 \cup B_2(x)$

2.$B_1 \cup B_2(x) = \max\{B_1(x), B_2(x)\}$
   $\leq \max\{B_1(x * y), B_2(y)\}, \max\{B_2(x * y), B_2(y)\}\}$
   $= \max\{B_1(x * y), B_2(x * y)\}, \max\{B_1(y), B_2(y)\}\}$
   $= B_1 \cup B_2(x * y), B_1 \cup B_2(x)(y)\}$

3.$B_1 \cup B_2(x * y) = \max\{B_1(x * y), B_2(x * y)\}$
   $\leq max\, \max\{B_1(x), B_2(y)\}, \max\{B_1(x), B_2(y)\}\}$
   $= max\, \max\{B_1(x), B_2(x)\}, \max\{B_1(y), B_2(y)\}\}$
   $= \max\{B_1 \cup B_2(x), B_1 \cup B_2(y)\}$

Hence the proof.

Definition 3.5:

Let B be a multi fuzzy set in a BH-algebra X. Then B is referred to as Anti multi fuzzy closed ideal in X if it satisfies the subsequent conditions:

1. $B(x) \leq \max\{B(x * y), B(y)\}$
2. $B(0 * y) \leq B(x)$

Example 3.6: Let $X = \{0,1,2,3\}$ be a set with the subsequent cayley table.

*	0	1	2	3
0	0	1	2	3

1	1	0	1	1
2	2	2	0	2
3	3	3	3	0

Let $B: X \rightarrow I$ be a multi-fuzzy set described with the aid of using $B(0) = B(1) = (0.6,0.8)$ and $B(2) = B(3) = (0.3,0.4)$.
Then B is known as multi-fuzzy closed ideal in X.

Theorem 3.7: Every Anti multi-fuzzy closed ideal is a Anti multi fuzzy ideal in X.
Proof: Let B be a Anti multi fuzzy closed ideal in X.
To show that B is a Anti multi fuzzy ideal in X
It is sufficient to show that $B(0) \leq B(x)$
Now , $B(0) \leq \max\{B(0 * x), B(x)\}$
Since through the use of Anti multi fuzzy closed ideal conditions
$B(0) \leq \max\{B(x), B(x)\}$
$= B(x)$
Clearly ii and iii are true.

Remark 3.8
The speak of the above theorem is not always true.

Theorem 3.9
If B is a Anti multi fuzzy BH-ideal in X, then the set $\cup\ (B; s)$ is a BH-ideal in X for $s = s_1, s_2, \ldots\ldots s_k)$ where $s_i \in [0,1]$, for all i.
Proof:
Let B be a Anti multi fuzzy BH-ideal in X.
To show that $\cup\ (B; s)$ is a BH-ideal in X
i) Since $B(0) \leq B(x) \leq s$
ii) Let $x * y \in U(B; s)$ and $y \in U(B, s)$
Then $B(x * y) \leq s$ and $B(y) \leq s$
Now B(x)$\leq \max\{B(x * y), B(y)\}$
$\leq \max\{s, s\} = s$
This implies that $x \in U(B; s)$
iii) Let $x \in U(B; s)$ and $y \in X$
Choose $y \in X$ such that $B(y) \leq s$
$B(x * y) \leq \max\{B(x), B(y)\}$
$\leq \max\{s, s\} = s$
This implies that $x * y \in U(B; s)$
Hence $U(B; s)$ is a BH- ideal in X.

4. HOMOMORPHISM OF ANTI MULTI-FUZZY BH-IDEALS :

In this segment we mentioned approximately the properties of Anti multi fuzzy BH-ideals under homomorphism.

Definition 4.1
Let $g: X \rightarrow Y$ be a mapping of BH-algebra and B be a Anti multi fuzzy set Y then $g^{-1}(B)$ is the pre-image of B under g if $g^{-1}(x) = B(g(x))\ \forall x \in X$.

Theorem 4.2

Let $g: X \to Y$ be a homomorphism of BH-algebra. If B is Anti multi fuzzy BH-ideal of Y. Then $g^{-1}(B)$ is a Anti multi fuzzy BH-ideal of X.

Proof:

Let $g: X \to Y$ be a homomorphism of BH-algebra.

Let B is a Anti multi fuzzy BH-ideal of Y.

To show that $g^{-1}(B)$ is a Anti multi fuzzy BH-ideal of X.

For any $x \in X$,

By the usage of Anti multi fuzzy BH-ideal.

1)$g^{-1}(B)(x) = B\big(g(x)\big) \leq B(0)$

$= B\big(g(0)\big)$

$= f^{-1}(B)(0)$

2)$g^{-1}(B)(x) = B\big(g(x)\big) \leq \max\big\{B\big(g(x)\big) * B\big(g(y)\big), B\big(g(y)\big)\big\}$

$= \max\{B(g(x * y), B\big(g(y)\big)\}$

$= \max\{g^{-1}(B)(x * y), g^{-1}(B)(y)\}$

3)$g^{-1}(B)(x * y) = B\big(g(x * y)\big) = B\big(g(x) * g(y)\big)$

$\leq \max\{B(g(x), B\big(g(y)\big)\}$

$= \max\{g^{-1}(B)(x), g^{-1}(B)(y)\}$

Hence $g^{-1}(B)$ is a Anti multi fuzzy BH-ideal of X.

Theorem 4.3

Let $g: X \to Y$ be an epimorphism of a BH-algebra. If $g^{-1}(B)$ is a Anti multi fuzzy ideal in X then B is a Anti multi fuzzy ideal in Y.

Proof:

Let $g: X \to Y$ be an epimorphism of a BH-algebra

Let $g^{-1}(B)$ is a Anti multi fuzzy ideal in X

To show that B is a Anti multi fuzzy ideal in Y.

Let $y \in Y$ there exists $x \in X$ such that $g(x) = yB(y)$

$= B\big(g(x)\big) = g^{-1}(B)(x)$

$\leq g^{-1}(B)(0)$

$= B\big(g(0)\big) = B(0)$

That is $B(0) \geq B(y)$

ii) Let $x, y \in Y$ there exists $a, b \in X$ such that $g(a) = x, g(b) = y$

$B(x) = B\big(g(a)\big)$

$= g^{-1}(B)(a)$

$\leq \max\{g^{-1}(B)(a * b), g^{-1}(B)(b)\}$

$= \max\big\{B\big(g(a * b)\big), B\big(g(b)\big)\big\}$

$= \max\big\{B\big(g(a) * g(b)\big), B\big(g(b)\big)\big\}$

$= \max\{B(x * y), B(y)\}$

iii)$B(x * y) = B\big(g(a) * g(b)\big)$

$= B\big(g(a * b)\big)$

$= g^{-1}(B)(a * b)$

$\leq \max\{g^{-1}(B)(a), g^{-1}(B)(b)\}$

$= \max\{B(g(a)), B(g(b))\}$

$= \max\{B(x), B(y)\}$

Hence B is Anti multi-fuzzy BH-ideal in Y.

REFERENCES:

[1]. Y. Imai and K. Iseki, On axiom system of propositional calculi, XIV Proc, Japan Academy, 42(1966) 19-22.

[2]. K. Iseki, An algebra related with a propositional calculi, Proc, Japan Acad 42(1966), 26-29

[3]. Iseki K: "On BCI-algebras, Math. Seminar Notes. 11(1980), 313-320

[4]. Iseki K and Tanaka S: " An introduction to theory of BCK-algebras", Math, Japan 23(1978), 1-26.

[5].L.A. Zadeh, Fuzzy sets, Information and Control, 8(1965) 338-353.

[6]. S. Sabu and T.V. Ramakrishnan, Multi-fuzzy sets, International Mathematical Forum, 50 (2010) 2471-2476.

[7]. Y.B. Jun, E.H. Roh and H.S. Kim, On BH-algebras, Scientiae Mathematicae 1(1) (1998), 347-354

[8]. Q. Zhang, E.H. Roh and Y.B. Jun, On Fuzzy BH-algebras, J. Huanggang Normal Univ, 21(3)(2001), 14-19.

[9]. K. Anitha and N. Kandaraj, Fuzzy subalgebras on BH-algebras, American International Journal of Research in Science, Technology, Engineering & Mathematics, (2019), 27-36

[10].K. Anitha and N. Kandaraj Fuzzy Ideals and Fuzzy Dot Ideals on BH-Algebras" International Journal of Advanced Research in Engineering and Technology, (2019) 359-361.

A NOTE ON β-$g\omega$-OPEN SETS

H.J. Saradha Devi[1] and N. Kandaraj[2]

[1] Research Scholar, PG and Research Department of Mathematics,
Saiva Bhanu Kshatriya College,
(Affiliated to Madurai Kamaraj University, Madurai)
Aruppukottai, Tamil Nadu, India.
E-mail: saradhadeviharikesavan@gmail.com

[2] Associate Professor, PG and Research Department of Mathematics,
Saiva Bhanu Kshatriya College
(Affiliated to Madurai Kamaraj University, Madurai),
Aruppukottai, Tamil Nadu, India.
Email:n.kandarajsbkc1998@gmail.com

ABSTRACT : *This paper introduces a new type of generalised ω-open sets, as well as some generalised locally closed sets in topological spaces, in order to derive a decomposition of ω-continuity.*

Key words: *$\beta - g\omega -$open set, locally closed set, $\beta - g\omega -$ locally closed set, locally $\omega - \beta$-closed set.*

1. INTRODUCTION:

An updated version of generalised closed sets was first presented by Levine [5] in the field of topology. In a topological space, Andrijevic [4] defined a category of generalised open sets called b-open sets. Gama-open sets are a subclass of sets that were studied by Ekici and Caldas [8]. In his 1980 paper, Dunham examined the topological findings of generalised closed sets. Many inquiries about b-open sets were explored by Ganster [7]. Since the inception of these ideas, several studies have been documented, each with its own unique set of findings.

This paper presents a new category of semi-generalized b-closed sets, semi-generalized b-open sets, Tsgb-space and investigates their connections to related classes. The differences between open sets and closed sets, as well as their qualities, have been explored. In addition, a new operator, the lorry operator, has been added, and some of its attributes have been investigated in this chapter.

2. SEMI GENERALIZED b-CLOSED SET :

Here, the definition of semi-generalized b-closed set and certain characterizations are discussed.

Definition 2.1: If $\mathrm{bCl}\,(A) \subseteq G$, then subset A of (X, τ) is consider to be a semi generalized $b - \text{closed set}$ represented by $\text{sgb} - \text{closed set}$ whenever G and $A \subseteq G$ is semi open in (X, τ).

Definition 2.2: The notation $\text{sgbc}(X)$ stands for the set of all $\text{sgb} - \text{closed sets}$ in the topological space (X, τ).

Theorem 2.3: Suppose (X, τ) contains A be a sgb − closed subset ,that means non-empty closed sets does not contain in bCl (A) − A.

Proof: Suppose $F \in Cl(X)$ such that $F \subseteq bCl(A) - A$ where $X - F$ is semi open.

$A \subseteq X - F$ and A will be sgb − closed.

Its follows that $bCl(A) \subseteq X - F$

Hence $F \subseteq X - bCl(A)$.

Which infers that $F \subseteq (X - bCl(A)) \cap (bCl(A) - A) = \varphi$.

Therefore $F = \varphi$.

Corollary 2.4: Suppose A be consider as sgb − closed set. Therefore iff bCl (A)− A is closed set then A is said to be b − closed.

Proof: Necessary part: Suppose A be a sgb − closed set.bCl (A) − A = φ which is closed set when A is b-closed.

Converse part: Consider bCl (A) − A be closed, here with theorem 4.2.3 bCl (A) − A will not have any non-empty closed subset and as bCl (A)− A will be closed subset of itself.

Then,

$$bCl(A) - A = \varphi$$

$$bCl(A) = A$$

and A is b − closed set.

Theorem 2.5: Suppose $B \subseteq A \subseteq X$ in which A is a semi-open set and sgb-closed set. B is then sgb-closed relation to A iff B is sgb − closed in X.

Proof: Necessary part: It is first considered that as B⊆A and A are both sgb-closed and semi open set, $bCl(A) \subseteq A$ and thus $bCl(B) \subseteq bCl(A) \subseteq A$

Hence, $A \cap bCl(B) = bCl_A(B)$

$$bCl(B) = bCl_A(B) \subseteq A$$

Given that B is sgb − closed with respect to A and G will be a semi-open subset of X,

$B \subseteq G$, therefore

$$B = B \cap A \subseteq G \subseteq A$$

In which $G \cap A$ is semi open in A.

That means, B is sgb − closed relative to A,

$$bCl(B) = bCl_A(B) \subseteq G \cap A \Rightarrow bCl(B) \subseteq G$$

Therefore B is sgb − closed in X.

Converse part: Suppose $B \subseteq G$, then $G = V \cap A$ for certain semi open subset V of X.

As $B \subseteq V$ and B is sgb − closed in X,

$bCl(B) \subseteq V$,

Hence $bClA(B) = bCl(B) \cap A \subseteq V \cap A \subseteq G. \Rightarrow bClA(B) \subseteq G$

Therefore B is sgb − closed in relation to A.

Remark 2.6: Suppose subset A be semi open and sgb-closed, $A \cap F$ is then sgb-closed whenever $F \in bCl(X)$.

Proof: The set A will be b-closed since $bCl(A) \subseteq A$ and A is sgb-closed and semi-open. As a result, $A \cap F$ is b-closed in X that indicates that $A \cap F$ is sgb-closed in X.

Hence A is semi open set and represented as −closed , thereore $bCl(A) \subseteq A$ and that means

A is b − closed. Therefore, A ∩ F is b-closed in X that infers that
A ∩ F will be sgb-closed in X.

Theorem 2.7: When A is a sgb-closed set and B is any set so that $A \subseteq B \subseteq bCl(A)$, B is then a sgb − closed set.

Proof: Suppose $B \subseteq G$ where G is semi open set. As A is sgb-closed set and $A \subseteq G$ therefore $bCl(A) \subseteq G$ and also $bCl(A) = bCl(B)$.

That means, $bCl(B) \subseteq G$ and thus B is sgb − closed set.

Theorem 2.8: Any pair of sets that intersect is also a sgb − closed set.

Proof: Suppose A and B be two sgb − closed set, that is, $bCl(A) \subseteq G$ whenever $A \subseteq G$ and G is semi open & $bCl(B) \subseteq G$ wherever $B \subseteq G$ and G is semi open.

Now, $bCl(A \cap B) = bCl(A) \cap bCl(B) \subseteq G$

Here $A \cap B \subseteq G$ and G is semi-open. Hence, any two sgb − closed sets may intersect inside themselves.

Remark 2.9: As given in the subsequent example, union of any two sgb − closed sets is not required to be a sgb − closed set.

Example 2.10: Suppose $X = \{a, b, c\}, \tau = \{X, \varphi, \{a, b\}\}$ in this topology space (X, τ), Therefore, the union of a and b does not constitute a sgb-closed set, despite the fact that the subsets a and b themselves do.

Theorem 2.11: Each b − closed set is sgb − closed set.

Proof: Let us assume that X includes two sets, A, which is a b-closed set, and G, which is a semi-open set that contains A**.** Therefore, each b − closed set is sgb − closed set.

Here $G \supseteq A = bcl(A)$

Remark 2.12: As given in the subsequent example, the reverse of the above theorem does not necessarily have to be correct.

Example 2.13: Suppose $X = \{a, b, c\}, \tau = \{X, \varphi, \{a\}, \{c\}, \{a, c\}\}$, this topological space (X, τ), the subset {a, c} will be sgb-closed which will not be b-closed set.

Theorem 2.14: Each swg − closed set is sgb − closed set.

Proof: Suppose A be a swg − closed set.

That means $Cl(Int\, A) \subseteq G$ in this equation.

$A \subseteq G$ & G are semi-open.

Due to the fact that all semi-closed sets are b-closed sets, $bCl\, A \subseteq Cl(Int\, A) \subseteq G$ and G is semi-open.

That means A is sgb − closed set.

Remark 2.15: As given in the subsequent example, the reverse of the above theorem does not necessarily have to be correct.

Example 2.16: Suppose $X = \{a, b, c\}, \tau = \{X, \varphi, \{b\}, \{c\}, \{b, c\}\}$. Subset {b} is a sgb − closed set, which is not a swg − closed set, in this topological space (X, τ).

Theorem 2.17: Each gα − closed set is sgb-closed set.

Proof: Suppose A be a gα − closed set therefore, $\alpha\, Cl\, A \subseteq G$ whenever $A \subseteq G$ and G is α − open. Hence, each α − closed sets are b-closed sets,

$bCl(A) \subseteq \alpha\, ClA \subseteq G$ and G is semi – open. Therefore A is sgb − closed.

Hence, each gα − closed set is sgb-closed set.

3. SEMI GENERALIZED b-OPEN SETS :

This section describes the characteristics of the newly discovered class of semi-generalized b-open set in topological spaces that has been introduced here.

Definition 3.1: When the complement A^c of a subset A of (X, τ) is also a semi generalised b-open set, we say that A is semi generalised b-open.

The set $sgbO(X)$ represents all sets in X that are sgb-open.

Theorem 3.2: A subset $A \subseteq X$ will be sgb-closed set if $F \subseteq b\ Int\ (A)$ if F is closed set and $F \subseteq A$.

Proof: Suppose A be a $sgb - open\ set$ & consider $F \subseteq A$ here F is closed, $X - A$ is that means a $sgb - closed$ set in the semi open set $X - F$. Therefore,
$bCl\ (X - A) \subseteq X - F$ and $X - b\ Int\ (A) \subseteq X - F$.Therefore $F \subseteq b\ Int\ (A)$. Contrariwise, if F is a closed set with $F \subseteq b\ Int\ (A)$ and $F \subseteq A$ therefore $X - b\ Int\ (A) \subseteq X - F$, that means $bCl\ (X - A) \subseteq X - F$.Therefore, $X - A$ is $sgb - closed\ set$ and A is a $sgb -$ closed set.

4. SEPERATION AXIOMS OF T_{sgb}-SPACES

In this section, we investigate the axiom-splitting process and develop a new kind of topological space called Tsgb-space. The connection to related areas is also elaborated upon.

Definition 4.1: (X, τ) is considered to be T_{sgb} – space with a condition that each $sgb -$ closed set is semi-closed set.

Theorem 4.2: Each T_{swg} – space is T_{sgb} – space.

Proof: Suppose X be T_{swg} – space and A be a $swg - closed\ set$ in X that means A is considered as $sgb - closed$ set through Theorem 4.2.14. Due to the fact that X is a T_{swg} – space, A is closed, and as a result, it is semi-closed. As a result, X belongs to the T_{sgb} – space class.

Remark 4.3: As may be seen from the following example, the reverse of the above theorem does not necessarily have to be correct.

Example 4.4: Suppose $X = \{a, b, c\}, \tau = \{X, \varphi, \{a\}\}$. In this topological space (X, τ) is $T_{sgb} -$ space and not T_{swg} -space, where the subset $\{b\}$ is $swg - closed$ which is not closed set.

Remark 4.5: Here are some illustrations that illustrate how independent the T_{sgb} – space and pre-$T_{1/2} -$ space are from one another.

Example 4.6: Suppose $X = \{a, b, c\}, \tau = \{X, \varphi, \{a\}\}$. In this topological space (X, τ) is $T_{sgb} -$ space and not pre $T_{1/2}$ -spaces, due to the fact that the subset $\{a, b\}$ is a gp-closed set, rather than a pre-closed set.

Example 4.7: Suppose $X = \{a, b, c\}, \tau = \{X, \varphi, \{a, b\}\}$.In this topological space (X, τ) is pre $T_{1/2} -$ space and not T_{swg} – spaces, since the subset {a} is sgb-closed set which is not semi-closed set.

Remark 4.8: The following examples demonstrate that the $T_{sgb} -$ spaces and $T_d -$ spaces are distinct from one another.

Example 4.9: Suppose $X = \{a, b, c\}, \tau = \{X, \varphi, \{a\}, \{a, b\}\}$. In this topological space (X, τ) is $T_{sgb} -$ spaces and not $T_d -$ spaces, since the subset $\{b\}$ is $gs -$ closed set that is not g-closed set.

Example 4.10: Suppose $X = \{a, b, c\}, \tau = \{X, \varphi, \{c\}, \{a, b\}\}$ this topological space (X, τ) is T_d − space and not T_{sgb} – spaces, since the subset $\{b\}$ is sgb − closed set which is not semi-closed set.

Definition 4.11: For any subset E of (X, τ), the following is defined,

$bCl^*(E) = \cap\ A : E \subset A(\in bD(X, \tau))$

Where $bD(X, \tau) = \{A : A \subset X$ and A is sgb closed in $(X, \tau)\}$

Theorem 4.12: Suppose E and F be the two subsets of a space (X, τ).Then,

(i) $E \subset bcl^*(E) \subset bcl(E) \subset cl(E)$

(ii) $bcl^*(\varphi) = \varphi$ and $bcl^*(X) = X$

(iii) $bcl^*(E \cup F) \supset bcl^*(E) \cup bcl^*(F)$

(iv) $bcl^*(bcl^*E) = bcl^*(E)$ and

(v) if E is sgb − closed then $bcl^*(E) = E$.

The argument may be shown to be self-evident once the definitions and characteristics of sgb − closed sets are understood.

Theorem 4.13: For every $x \in X, \{x\}$ will be semi-closed or its compliment $\{x\}^{\{c\}}$ will be sgb-closed in a space (X, τ).

Proof: Consider $\{x\}$ is not semi-closed in (X, τ).As $\{x\}^{\{c\}}$ will not be semi- open. The space X itself is only semi-open set containing $\{x\}^{\{c\}}$. Therefore, $bCl(\{x\}^{\{c\}})$ holds and $\{x\}^{\{c\}}$is sgb-closed.

Theorem 4.14: For a space (X, τ) if $x \neq y$ then $bcl^*(x) \neq bcl^*(y)$.

Proof: With the help of above Theorem, it is sufficient to prove the following, that is

$\{x\}^{\{c\}}$ is sgb − closed.Since $\{y\} \subset \{x\}^{\{c\}}, y \in bcl^*(\{y\}) \subset \{x\}^{\{c\}}$,

$bcl^*(\{y\}) \neq bcl^*(\{x\})$.

Definition 4.15: $S.O\ (X, \tau)^* = \{B : bcl^*(B^c) = (B^c)\}$

Remarks 4.16: If $E \in bD(X, \tau)$ (Def. 4.5.1) then $E^c \in S.O(X, \tau)^*$

Theorem 4.17: (i) $S.O.(\tau) \subset S.O.(\tau)^*$ holds

(ii) A space (X, τ) is T_{sgb} if and only if $S.O.(\tau) \in S.O.(\tau)^*$ holds.

Proof: (i) Suppose $E \in S.O.(\tau)$, its complement E^c then is semi-closed, if and only if $E^c = bCl(E^c)$, which follows from Theorem 4.5.2 (i) that $bcl^*(E^c) = E^c$ holds.

That is $E \in S.O.(\tau)^*$.

(ii) **Necessity**: Given that the assumption is true, the semi-closed sets and the sgb − closed sets are equivalent, $bCl(E) = bcl^*(E)$ holds for every subset of (X, τ).

Hence $S.O.(\tau) \in S.O.(\tau)^*$

Sufficiency: Suppose A be as gb − closed set of (X, τ).

With the help of Theorem 4.5.2(V), then $A = bcl^*(A)$ and hence $A^c \in S.O.(\tau)$

Hence A is semi-closed. That means $(X, \tau)$is T_{sgb} − space.

REFERENCES:

[1] Maki H, Sundram P, Balachandran K. On generalized continuous maps in topological spaces. Memoirs of Faculty of Science Kochi University Series A Mathematics 1991; 12: 5-13.

[2] El-Monsef MEA, El-Deeb SN, Mahmoud RA. β-open sets and β-continuous mappings.

Bulletin of Faculty of Science Assiut University. 1983; 12(1): 77- 90.
[3] Njastad O. On some classes of nearly open sets. Pacific Journal of Mathematics. 1965; 15(3): 961-970.
[4] Andrijevic D. Semi-preopen sets. MatematickiVesnik. 1986; 38 (93): 24-32.
[5] Levine N. Generalized closed sets in topology. RendicontidelCircoloMatematico di Palermo. 1970; 19(2): 89-96.
[6] Benchalli SS, Patil PG, Rayanagaudar TD. ωα-Closed sets in topological Spaces. The Global Journal of Applied Mathematics and Mathematical Sciences. 2009; 2:53-63.
[7] Ganster. M,Reilly. I. L,"Locally closed sets and alc- continuous functions", InternatJ.Math.Math.Sci.3(1989), 417-424.
[8] Ekici E, Caldas M, Slightly γ-continuous functions, Boletim da Sociedade Paranaense de Matematica, 2004, 22(2), pp 63-74.